GARSIDE FAMILIES, GARSIDE MONOIDS AND CATEGORIES

The first example of a group with a Garside structure is the braid group, seen as the group of fractions of the monoid of positive braids. After many extensions to other groups and monoids, Garside structures were axiomatized and then generalized to categories and groupoids.

The main feature of a Garside structure in a monoid or a category is the existence of a subset called a Garside family with respect to which any element has a "greedy" normal form. This stems from the properties of left or right divisibility. I will give the basic axioms and the main properties of these normal decompositions.

When the Garside family is bounded (existence of a Garside element in the monoid case or of a Garside functor in the categorical background), the properties extend to the enveloping group (or groupoid). I will explain the example of the conjugation category in a Garside category or monoid.

The main sources are

- the paper *Garside families and Garside germs* by Dehornoy, Digne and Michel in J. Algebra 380 (2013).
- Section I of the paper *Parabolic Deligne-Lusztig varieties* by Digne and Michel in Adv. Math. 257 (2014)
- Chpapters III to V of the book by Dehornoy, Digne, Godelle, Krammer, Michel, *Foundations of Garside theory*, EMS Tracts in Math. 22 (2015)