COHEN-MACAULAY MODULES IN REPRESENTATION THEORY

BIREP SUMMER SCHOOL 2019

1. Abstract

The study of Cohen-Macaulay modules connects representation theory with many other areas such as commutative algebra, singularity theory and physics. CM-finiteness is one of the fundamental problems. Classically, this has been studied for lattices over orders and hypersurface singularities. A powerful tool to visualize, analyze and understand the category of Cohen-Macaulay modules for (non-commutative) isolated singularities, in particular in all CM-finite cases, is provided by Auslander-Reiten theory. The stable category of Cohen-Macaulay modules over a Gorenstein ring is our prototype of a triangulated category. In results of Buchweitz and Orlov it is linked to algebraic geometry and in results of Iyama and others to cluster theory.

2. Classical orders of CM-finite type

In **Talk 2–5** you may assume for simplicity $R = k[[x]]$ for some field k and AR theory should be used without a proof. It will be explained properly in talk **Talk 9:**.

Talk 1: R-orders and Krull-Schmidtness (45 min)
Let R be a commutative noetherian ring and Λ a module-finite R-algebra such that R is a central subring. If R is a complete local ring, then the category Λ-mod of finitely generated Λ-modules is a Krull-Schmidt category (cf. [LW12, Chapter 1.2], [CR90, §6B]). If R is also regular (think of $R = K[[x_1, \ldots, x_d]]$ for a field k), then Λ is called an **R-order** if ${}_R\Lambda$ is a free R-module and we define the full subcategory of Λ-lattices (or Cohen-Macaulay Λ-modules) $\mathrm{CM}(\Lambda)$. Please explain AR theory in this situation following [Iya08, section 3.1].

Talk 2: Maximal and hereditary orders
This talk deals with maximal and hereditary orders over R. We refer [CR90, §26]) for definitions and results of maximal and hereditary orders. Explain the structure theorem of these orders, see also [Iya01, Subsection 1.3]. There exists a characterization of hereditary orders in terms of their CM categories, see [HN94, 1.6 Theorem]. At the end, the rejection lemma by Drozd-Kirichenko for complete discrete valuation rings should be discussed, see [HN94, p. 2.2.1].

Talk 3: Backstrom orders
In [RR79], Backstrom orders over a complete valuation ring were studied. First theorem of this orders is an equivalence between the category of CM modules and a subcategory of the module category over some artin algebra. By using this theorem together with Ringel's characterization of representation finiteness of valued graphs, the characterization of CM-finite Backstrom orders was given. In [Rog84], the Auslander-Reiten species of Backstrom orders are given. Ribbon graph orders are a very nice example of Backstrom orders (preprint by W. Gnedin is to appear).

Talk 4: Tiled orders (optional)
For a discrete valuation ring R, tiled orders over R are orders in matrix algebras over the fractional field of R. Explain a characterization of CM-finite tiled orders by [ZK77]. There is a characterization of tame Cohen-Macaulay type and of polynomial growth tiled orders when R is the power series algebra in one indeterminate, see [Sim00, Sections 7, 8].

Talk 5: Bass orders (optional)
Bass orders are a fundamental class of Gorenstein orders of CM-finite type. Two papers by Hijikata and Nishida [HN92; HN94] give a classification result of Bass orders. We can see AR quivers of Bass orders in [Iya01, Section 4].

Talk 6: Commutative CM-finite type of dimension zero and one

In this talk, commutative CM-finite rings of Krull dimension zero and one will be discussed. We refer to [LW12, Chapter 3] for the dimension zero case. In the dimension one case, a necessary and sufficient condition is known, see [LW12, Chapter 4]. For ADE singularities of dimension one, we can see AR quivers of the category of CM modules in [Yos90, Chapter 9].

Talk 7: Commutative ring theory

Basic concepts of commutative ring theory are given in this talk, which will be used in the following talks. For a commutative local noetherian ring, define the depth of moduSebastian Opper les and Cohen-Macaulay modules, cf. [Iya08] beginning of Subsection 3.1. Then regular, Gorenstein and Cohen-Macaulay rings should be defined. The Auslander-Buchsbaum formula (cf. Theorem 1.3.3 in [BH93]) and Serre's homological characterization of regular rings (cf. Theorem 19.12 in [Eis95]) are fundamental results. Observe that for a Cohen-Macaulay local ring with a canonical module there is a nice duality between its category of Cohen-Macaulay modules (cf. Theorem 21.21 in [Eis95]). We refer to [BH93; LW12; Mat86].

Talk 8: Auslander-Reiten theory for lattices - I (45 min)

In general for a commutative Cohen-Macaulay ring R and a module-finite algebra Λ, with R is central and Λ is a max CM R-module, we always denote by $\mathrm{CM}(\Lambda) := \{\mathrm{X} \in \Lambda\text{-mod} \mid \mathrm{X} \in (\max)\ \mathrm{CM(R)}\}$. Define an R-order Λ and the category of Λ-lattices $L(\Lambda)$ following [Aus78], Section 7, Chapter I[1]. The aim of this talk is to explain AR duality, Section 7 and Section 8 of Chapter I of [Aus78], more precisely look at Propositions 7.10 and 8.17. (Please also read the last paragraph of the appendix of [Aus86a].) AR duality induces a Serre functor on the singularity category (cf. later talk **Talk 20:**).

Talk 9: Auslander-Reiten theory for lattices - II (45 min)

In this talk Auslander-Reiten theory and the existence of almost split sequences for the category of CM modules are explained. Define almost split sequences in the category and define an order to be an non-singular and an isolated singularity (cf. [LW12, Chapter 13], [Iya08, §3.1]). Isolated singularities are characterized by finiteness of length of morphism spaces (commutative: [LW12; Yos90]; non-commutative: [Aus86a; AR87] and [Iya08, Subsection 3.1]). Explain that an order is an isolated singularity if and only if the category of CM modules has almost split sequences (the converse is only proven for R regular local complete in [Aus86a]). Observe that for R regular local complete, CM finiteness implies isolated singularity.

Talk 10: Auslander-Buchweitz approximations

The talk contains Auslander-Buchweitz's theory of CM approximations (cf. [Col89, Theorem 1.1 and Thm B], loc. cit. Example 3 on p.14 and loc. cit. Example 4 on p.22 or alternatively [Buc, Theorems 5.1.2 and 5.1.4, §5.4]). In particular explain: The approximations are unique in the stable category.

Talk 11: Two-dimensional tame and maximal orders (optional)

In this talk AR-theory for $\dim R = 2$ should be explained, the main source is [RV89, Chapter 2 and Chapter 5][2]. Tame orders of finite representation-type are reflexive Morita equivalent to one of gldim 2 (cf. Proposition 1.2 and beginning of Chapter 2). To classify such orders, they used associated graded orders (see, Chapter 5, Corollary 5.7). In Chapter 2, they first describe the AR quiver of such a tame order. By using the AR quiver, they determine the Gabriel quiver and its relations for the associated graded order.

Talk 12: Algebraic McKay correspondence

Based on Auslander's original work [Aus86b], invariant subrings of the polynomial ring and skew group rings are studied. We refer to [Yos90, Chapter 10] for dimension two and [LW12, Chapter 5] for arbitrary dimension. We can see a short summary in [Iya18, Example 2.25].

[1] This subcategory of $\mathrm{CM}(\Lambda)$ is also denoted by $L_p(\Lambda)$ ($\neq L_P(\Lambda)$!!) or $\mathrm{CM}_0(\Lambda)$ in other sources. It coincides with $\mathrm{CM}(\Lambda)$ for an isolated singularity

[2] This reference is not appropriate for beginners

Talk 13: Knörrer's periodicity and hypersurface singularities
This talk follows [LW12, Chapter 8]. Define matrix factorizations of hypersurface singularities. Matrix factorizations and double branched covers give an important insight for CM-finite hypersurface singularities. Knörrer's periodicity reduces CM-finiteness of higher dimensional hypersurfaces to lower ones. Observe that CM-finiteness of hypersurfaces implies simple surface singularities, and therefore such hypersurfaces are given by polynomials of ADE type, which already appeared in the previous talk in the dimension one case.

Talk 14: CM-finiteness of dimension two and scrolls (optional)
Under some conditions, two dimensional CM-finite rings are isomorphic to invariant subrings of the polynomial ring [LW12, Chapter 7]. On the other hand, Auslander and Reiten gave a CM-finite ring which is not a hypersurface and its dimenion is greater than two [AR89], [Yos90, Chapter 16].

Talk 15: CM-Auslander correspondence (optional)
Explain [Eno17, Theorem C] and how it applies to the CM-finite R-orders which we considered in earlier talks (i.e. try to describe the CM-Auslander algebras, what are the idempotents e which one has to choose etc.).
According to the taste of the speaker: Alternatively explain the more general Auslander correspondence of type (d, d, n) from [Iya07a, Theorem 4.2.3] (the proof is explained in Subsection 4.6 but uses very general results) - Enomoto's result is the $(d, d, 1)$ correspondence. Also the $(d, d, d-1)$ case is peculiar (see loc. cit. Subsection 4.7).

3. The stable category of CM modules

As a reminder or a first introduction of the derived catgeory and Verdier quotients, we will organise a short meeting informally during the summer school.

Talk 16: Buchweitz's Theorem
Define the stable category of a Frobenius category and explain its triangulated structure following e.g. [Hap88, Chapter I]. Briefly review the definition and the triangulated structure of the bounded derived category and introduce the singularity category $\mathrm{D}^{\mathrm{b}}_{\mathrm{sg}}(\Lambda)$. Now Buchweitz's result [Buc, Theorem 4.4.1] should be proven, which establishes a triangle equivalence between the singularity category $\mathrm{D}^{\mathrm{b}}_{\mathrm{sg}}(\Lambda)$ and the stable category of Cohen-Macaulay modules $\underline{\mathrm{CM}}(\Lambda)$ for (Iwanaga-)Gorenstein rings Λ.

Talk 17: Orlov's Theorem
Introduce the categories $\mathrm{qgr}(\Lambda)$ for graded noetherian rings Λ following [Orl09, §2] or [BS15, §4]. For a motivation you might refer to [Ser55] (cf. also [BS15, Remark 4.2]). Discuss semiorthogonal decompositions for triangulated categories and define the graded singularity category $\mathrm{D}^{\mathrm{gr}}_{\mathrm{sg}}(\Lambda)$. After this, proceed with Orlov's Theorem [Orl09, Theorem 2.5] for positively graded noetherian Gorenstein rings Λ, which describes a fully faithful embedding between the triangulated categories $\mathrm{D}^{\mathrm{gr}}_{\mathrm{sg}}(\Lambda)$ and $\mathrm{D}^{\mathrm{b}}(\mathrm{qgr}(\Lambda))$ whose direction depends on the Gorenstein parameter of Λ. For (more general) expositions of the theorem see also Iyama-Yang [Iya18, Corollary 2.6] and [BS15, Theorem 6.4]. Include some of the examples given at the end of [Orl09, §2.2] and of those found in [BS15, §7].

Talk 18: Tilting objects for self-injective graded algebras (optional)
Generalizing Happel's triangle equivalence [Hap88] between the derived category $\mathrm{D}^{\mathrm{b}}(\mathrm{mod}(\Lambda))$ and the stable module category $\underline{\mathrm{mod}}^{\mathbb{Z}}(T(\Lambda))$ of the trivial extension $T(\Lambda)$ of a finite-dimensional algebra Λ of finite global dimension, Yamaura showed for positively graded self-injective finite-dimensional algebras A that $\underline{\mathrm{mod}}^{\mathbb{Z}}(A)$ admits a tilting object if and only if A_0 has finite global dimension. Explain the proof of this result [Yam13, Theorem 1.3]. Discuss the applications to higher preprojective algebras [Yam13, §4].

Talk 19: Tilting objects for graded Gorenstein rings of dimension one
Buchweitz, Iyama and Yamaura [BIY18, Theorems 1.3, 1.6 and 1.8] explicitly constructed under

certain assumptions a tilting object in the stable category of graded Cohen-Macaulay modules for commutative positively graded Gorenstein rings of dimension one.

4. Cluster-tilting for stable CM modules

Talk 20: Quotient singularities – ungraded setting (45 min)
Show that (some Gorenstein) quotient singularities fulfill that their stable category of Cohen Macaulay modules are $[d-1]$-Calabi-Yau categories with a cluster tilting object [IY08, Section 8].

Talk 21: Quotient singularities – graded setting (45 min)
Show that (some Gorenstein) quotient singularities fulfill that their stable category of graded Cohen Macaulay modules are $(-d)[d-1]$-Calabi-Yau categories with a tilting object [IT13, Theorems 1.5, 1.6 and 1.7].

Talk 22: Cyclic quotient singularities
In the case of a cyclic quotient singularity, one can identify the stable category of Cohen Macaulay modules with a generalized cluster category. The goal of this talk is to explain Section 5 of [AIR15] (in particular Theorem 5.1). The technical notion (of a bimodule d-Calabi-Yau algebra) of Sections 3 and 4 should be used as a black box- preprojective algebras should also be kept to a minimum. As motivation the McKay correspondence discussion from the introduction of loc. cit. (cf. also earlier talk) should serve.

Talk 23: Orders arising from triangulations of polygons
This talk is based on [DL16b]. From a given polygon and its triangulations, we can construct an ice quiver with potential such that its frozen part is a Gorenstein $K[X]$-order. We can classify all lattices over the order, and the order is of finite CM type. We can also see that the stable category of the lattices are equivalent to the cluster category of path algebras of Dynkin type A. Moreover, the stable category of graded lattices are equivalent to the derived category of path algebras of Dynkin type A.

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