

Working group: tt-geometry

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If not indicated otherwise, we are meeting in V5-227 at Bielefeld University at 4pm. The dates are written next to the speakers in the program.

General resources include Balmer's article [Bal19] for the Handbook of Homotopy Theory and Stevenson's lecture notes [Ste18].

1. **Balmer spectrum:** (Eike Lau, 18.11.20)

- Definition
- Examples: derived category of perfect complexes of modules over a commutative noetherian ring, stable module category for modules over the group algebra of a finite group, ...
- Classification Theorem

References include [Bal05, Bal19].

2. **Theorem of Hopkins-Neeman:** (Janina Letz, 18.11.20)

- State and prove the Theorem of Hopkins-Neeman
- Explain the connection to the Balmer spectrum and state Thomason's extension

References include [Nee92, Tho97], several proofs are sketched in Iyengar's extended abstract in [Iye06].

3. **Harish-Chandra modules:** (Fabian Januszewski, 02.12.20)

- Definitions over general fields (characteristic 0), homological properties, outline of constructions
- Exotic monoidal structures by Gal Dor

A reference for the algebraic theory of Harish-Chandra modules is [KV95].

4. **Monoidal structures on Harish-Chandra modules:** (Fabian Januszewski, 06.01.21, at 5pm online)

- Standard symmetric monoidal structure on (\mathfrak{g}, K) -modules
- Algebraic (homological) characters.

- Exotic monoidal structures by Gal Dor

A reference for exotic monoidal structures is [Dor20].

5. **Stratification of triangulated categories:** (Marc Stephan, 13.01.21, at 5pm online)

- Support
- Local-global principle
- Stratification of an R -linear triangulated category
- Classification theorems for localizing subcategories and tensor ideal localizing subcategories

References include [BIK11, BIK12].

6. **Stratification of representations of finite groups:** (Henning Krause, 27.01.21, at 5pm online)

- State the classification theorem for the localising subcategories of the large stable module category
- Proof in characteristic 2 or approach via π -points
- Explain connection to Balmer spectrum

References include [BIK11, BIK12, BIKP17].

7. **\mathbb{A}^1 over monoidal derivators:** (Tobias Columbus, 10.02.21, at 5pm online)

Describe the construction of $D(R[T])$ in terms of the derivator \mathbb{D}_R due to Balmer and Zhang and explain the construction of a "correct" tensor product on $D(R[T])$ in terms of the derivator \mathbb{D} .

A reference for the construction and motivation is [BZ17].

8. **Perfect chain complexes with small homology over $\mathbb{F}_2[A_4]$:** (Marc Stephan, 24.02.21, at 5pm online)

References include [BC87, BC94].

9. **Discussion about research problems:** (10.03.21, at 5pm online)

10. **The frame of alpha-localizing ideals and its connection to the tt-spectrum via Stone duality:** (Janina Letz, 28.10.21, at 4pm in U2-217)

A reference is [KP17].

11. **The stable derived category of a noetherian scheme:** (Henning Krause, 08.11.21, at 4pm in V4-112)

A reference is [Kra05].

12. **Finite permutation resolutions:** (Marc Stephan, 08.11.21, at 5pm in V4-112)
References include [BG20].
13. **Permutation modules and cohomological singularity:** (Eike Lau, 09.12.21, at 5.30pm in V2-200)
References include [BG21a].
14. **Artin motives I:** (Tobias Columbus, 20.12.21, at 4pm in V4-112)
References include [BG21b, BG19].
15. **Artin motives II:** (Fabian Januszewski, 20.12.21, at 5pm in V4-112)
References include [BG21b, BG19].
16. **Some aspect of permutation modules, Mackey functors and Artin motives:** (Tobias Columbus, 26.04.22, at 5pm in H6)
A reference is [BG21b].
17. **On the Noetherianness and connectedness of Balmer spectra of stable module categories of infinite groups:** (Rudradip Biswas, 19.05.22, at 4pm in H11)
18. **The spectrum of a well-generated tensor triangulated category:** (Janina Letz, 19.05.22, at 5pm in H11)
A reference is [KL22].
19. **Fibrewise stratification of group representations:** (Henning Krause, 24.05.22, at 4pm in T2-149)
A reference is [BIKP22].

References

- [Bal05] Paul Balmer, *The spectrum of prime ideals in tensor triangulated categories*, *J. Reine Angew. Math.* **588** (2005), 149–168. MR 2196732
- [Bal19] Paul Balmer, *A guide to tensor-triangular classification*, 2019, <https://arxiv.org/abs/1912.08963>.
- [BC87] David J. Benson and Jon F. Carlson, *Complexity and multiple complexes*, *Math. Z.* **195** (1987), no. 2, 221–238. MR 892053
- [BC94] D. J. Benson and Jon F. Carlson, *Projective resolutions and Poincaré duality complexes*, *Trans. Amer. Math. Soc.* **342** (1994), no. 2, 447–488. MR 1142778
- [BG19] Paul Balmer and Martin Gallauer, *Three real artin-tate motives*, 2019, <https://arxiv.org/abs/1906.02941>.

- [BG20] ———, *Finite permutation resolutions*, 2020, <https://arxiv.org/abs/2009.14091>.
- [BG21a] ———, *Permutation modules and cohomological singularity*, 2021, <https://arxiv.org/abs/2009.14093>.
- [BG21b] ———, *Permutation modules, Mackey functors, and Artin motives*, <https://people.maths.ox.ac.uk/gallauer/PmMfAm.pdf>, 2021, accessed 23 March 2021.
- [BIK11] David J. Benson, Srikanth B. Iyengar, and Henning Krause, *Stratifying modular representations of finite groups*, *Ann. of Math. (2)* **174** (2011), no. 3, 1643–1684. MR 2846489
- [BIK12] David J. Benson, Srikanth Iyengar, and Henning Krause, *Representations of finite groups: local cohomology and support*, *Oberwolfach Seminars*, vol. 43, Birkhäuser/Springer Basel AG, Basel, 2012. MR 2951763
- [BIKP17] Dave Benson, Srikanth B. Iyengar, Henning Krause, and Julia Pevtsova, *Stratification and π -cosupport: finite groups*, *Math. Z.* **287** (2017), no. 3-4, 947–965. MR 3719521
- [BIKP22] Dave Benson, Srikanth B. Iyengar, Henning Krause, and Julia Pevtsova, *Fibrewise stratification of group representations*, 2022, <https://arxiv.org/abs/2204.10431>.
- [BZ17] Paul Balmer and John Zhang, *Affine space over triangulated categories: a further invitation to Grothendieck derivators*, *J. Pure Appl. Algebra* **221** (2017), no. 7, 1560–1564. MR 3614966
- [Dor20] Gal Dor, *Exotic monoidal structures and abstractly automorphic representations for $GL(2)$* , 2020, <https://arxiv.org/abs/2011.03313>.
- [Iye06] *Mini-workshop: Thick Subcategories—Classifications and Applications*, vol. 3, 2006, Abstracts from the mini-workshop held February 19–25, 2006, Organized by Ragnar-Olaf Buchweitz, Henning Krause and Stefan Schwede, *Oberwolfach Reports*. Vol. 3, no. 1, pp. 461–509. MR 2278894
- [KL22] Henning Krause and Janina C. Letz, *The spectrum of a well-generated tensor triangulated category*, 2022, <https://arxiv.org/abs/2203.03249>.
- [KP17] Joachim Kock and Wolfgang Pitsch, *Hochster duality in derived categories and point-free reconstruction of schemes*, *Trans. Amer. Math. Soc.* **369** (2017), no. 1, 223–261. MR 3557773
- [Kra05] Henning Krause, *The stable derived category of a Noetherian scheme*, *Compos. Math.* **141** (2005), no. 5, 1128–1162. MR 2157133

- [KV95] Anthony W. Knapp and David A. Vogan, Jr., *Cohomological induction and unitary representations*, Princeton Mathematical Series, vol. 45, Princeton University Press, Princeton, NJ, 1995. MR 1330919
- [Nee92] Amnon Neeman, *The chromatic tower for $D(R)$* , *Topology* **31** (1992), no. 3, 519–532, With an appendix by Marcel Bökstedt. MR 1174255
- [Ste18] Greg Stevenson, *A tour of support theory for triangulated categories through tensor triangular geometry*, Building bridges between algebra and topology, Adv. Courses Math. CRM Barcelona, Birkhäuser/Springer, Cham, 2018, pp. 63–101. MR 3793858
- [Tho97] R. W. Thomason, *The classification of triangulated subcategories*, *Compositio Math.* **105** (1997), no. 1, 1–27. MR 1436741