Ninth Workshop on Random Dynamical Systems

14–17 June 2017

Department of Mathematics
University of Bielefeld
Room V2–210/216

This workshop is part of the conference programme of the DFG-funded CRC 701
Spectral Structures and Topological Methods in Mathematics
at the Faculty of Mathematics, University of Bielefeld

Organizers: Wolf-Jürgen Beyn and Barbara Gentz (Bielefeld)

http://www.math.uni-bielefeld.de/~gentz/pages/SS17/RDS17/RDS17.html

Programme

Wednesday, 14 June 2017

09:45 – 10:20	*Registration and coffee in V3–201*
10:20 – 10:30	**Friedrich Götze** (Speaker of the CRC 701) *Opening of the workshop*
10:30 – 11:30	**Michael Scheutzow** (TU Berlin) Minimal random attractors
11:30 – 12:30	**Zdzisław Brzeźniak** (University of York) Stochastic Landau–Lifshitz–Bloch equation
12:30 – 14:00	*Lunch*
14:00 – 15:00	**Konstantinos Spiliopoulos** (Boston University) Hypoelliptic multiscale Langevin diffusions: large deviations, invariant measures and small mass asymptotics
15:00 – 15:30	**Christian Wiesel** (Universität Bielefeld) Noise-induced synchronization in a ring of weakly coupled oscillators
15:30 – 16:00	*Coffee in V3–201*
16:00 – 16:30	**Maximilian Engel** (Imperial College London) Quasi-stationary dynamics and bifurcations of random dynamical systems
16:30 – 17:00	**Christian Bayer** (WIAS Berlin) Numerics for rough volatility models
17:00 – 18:00	**Ľubomír Baňas** (Universität Bielefeld) Numerical approximation of the stochastic Cahn–Hilliard equation near the sharp interface limit

All talks will take place in V2–210/216.

Thursday, 15 June 2017

9:00 – 9:15	*Coffee in V3–201*
9:15 – 10:15	**Ellen Baake** (Universität Bielefeld) Solving the recombination equation
10:15 – 10:45	**Martin Redmann** (WIAS Berlin) A regression method to solve parabolic rough PDEs
10:45 – 11:15	*Coffee in V3–201*
11:15 – 12:15	**David Siska** (University of Edinburgh) L^p-estimates and regularity for SPDEs with monotone semi-linear term
12:15 – 15:00	*Lunch excursion**
15:00 – 16:00	**Erika Hausenblas** (Montanuniversität Leoben) The nonlinear Schrödinger Equation driven by Levy noise
16:00 – 16:30	**Isabell Vorkastner** (TU Berlin) Connectedness of random attractors
16:30 – 17:30	**Raphael Kruse** (TU Berlin) On randomized one-step methods for non-autonomous (stochastic) differential equations with time-irregular coefficients
all day	*Exhibition in V2–213* *Women of Mathematics throughout Europe — A gallery of portraits*

* Please note:

Thursday is a public holiday (Corpus Christi) in the state of North Rhine-Westphalia. Mensa and all food stalls on campus will be closed. Therefore, we have arranged for a Turkish-style lunch buffet off campus at Uni-Eck. Please join the group for the short walk there.

Uni-Eck, Wertherstr. 255a, 33619 Bielefeld, ☎ +49 521 988 39 801

All talks will take place in V2–210/216.

Friday, 16 June 2017

9:00 – 9:15	*Coffee in V3–201*
9:15 – 10:15	**Fabienne Castell** (Aix–Marseille Université) Multiresolution analysis on graphs using random forests and Markov process intertwining
10:15 – 10:45	**Andreas Rößler** (Universität zu Lübeck) Numerical methods for stochastic differential–algebraic equations — some recent developments
10:45 – 11:15	*Coffee in V3–201*
11:15 – 12:15	**Sotirios Sabanis** (University of Edinburgh) Recursive estimators and MCMC algorithms
12:15 – 14:00	*Lunch*
14:00 – 15:00	**Grégory Maillard** (Aix–Marseille Université) Parabolic Anderson model in a dynamic random environment: random conductances
15:00 – 15:30	**Alexandra Neamtu** (Friedrich-Schiller-Universität Jena) On the Oseledets splitting for infinite-dimensional random dynamical systems
15:30 – 16:30	**Sebastian Riedel** (TU Berlin) Random dynamical systems and rough paths
16:30 – 17:15	*Tea in V3–201*
17:15 – 18:15	*Mathematical Colloquium of the Faculty of Mathematics* **Vojkan Jakšić** (McGill University) Time and entropy
19:30 –	*Joint dinner in the city centre* Wernings Weinstube Alter Markt 1, 33602 Bielefeld, ☎ +49 521 136 51 51 http://www.wernings-weinstube.de Please note: For the dinner, prior registration is required.

All talks will take place in V2–210/216.

Saturday, 17 June 2017

9:00 – 9:15	*Coffee in V3–201*
9:15 – 10:15	**Felix Lindner** (TU Kaiserslautern) Strong approximation of constrained stochastic dynamics
10:15 – 10:45	**Julian Newman** (Universität Bielefeld) Topological conjugacy of iterated random homeomorphisms of the circle
10:45 – 11:15	*Coffee in V3–201*
11:15 – 12:15	**Martin Kolb** (Universität Paderborn) Local asymptotics for the area of random walk excursions
12:15 – 13:15	**Nils Berglund** (Université d'Orléans) Metastability in stochastic Allen–Cahn PDEs
13:15 – 14:30	*Lunch at the Westend-Cafeteria (next to the indoor pool)*

All talks will take place in V2–210/216.

Abstracts

Ellen Baake (Universität Bielefeld)

Solving the recombination equation

The recombination equation is a well-known dynamical system from mathematical population genetics, which describes the evolution of the genetic composition of a population that evolves under recombination. The genetic composition is described via a probability distribution (or measure) on a space of sequences of finite length; and recombination is the genetic mechanism in which two parent individuals are involved in creating the mixed sequence of their offspring during sexual reproduction. The model comes in a continuous-time and a discrete-time version; it can accommodate a variety of different mechanisms by which the genetic material of the offspring is partitioned across its parents. In all cases, the resulting equations are nonlinear and notoriously difficult to solve. Elucidating the underlying structure and finding solutions has been a challenge for a century.

In this talk, we show how the equation can be solved, in two ways: forward in time, via a modern version of so-called Haldane linearisation; and backward in time via an associated stochastic fragmentation process. This is joint work with Michael Baake.

Reference: E. Baake, M. Baake, *Haldane linearisation done right: Solving the recombination equation the easy way*, Discr. Cont. Dyn. Syst. A 36 (2016), 6645–6656.

Ľubomír Baňas (Universität Bielefeld)

Numerical approximation of the stochastic Cahn–Hilliard equation near the sharp interface limit

We present an implicit semi-discrete numerical scheme for the stochastic Cahn–Hilliard equation with (asymptotically small) noise term. For the proposed scheme we derive strong convergence rates which depend polynomially on the interfacial thickness parameter. In addition, we perform numerical studies to show asymptotic behavior of the model.

This is a joint work with D. Antonopoulou (Chester) and A. Prohl (Tübingen).

Christian Bayer (WIAS Berlin)

Numerics for rough volatility models

Markov processes are a very convenient, but restrictive modeling framework, in particular since a variety of different numerical methods are available. However, sometimes introducing auto-correlated noise can lead to substantial improvements in models. For instance, stochastic volatility models with very simple structure, but rough, fractional volatility can lead to extremly good fit to market prices of options. We present variations of rough volatility models and discuss numerical algorithms used for approximating prices in that setting.

Nils Berglund (Université d'Orléans)

Metastability in stochastic Allen–Cahn PDEs

Stochastic processes subject to weak noise often show a metastable behaviour, meaning that they converge to equilibrium extremely slowly; typically, the convergence time is exponentially large in the inverse of the variance of the noise (Arrhenius law).

In the case of finite-dimensional Ito stochastic differential equations, the large-deviation theory developed in the 1970s by Freidlin and Wentzell allows to prove such Arrhenius laws and compute their exponent. Sharper asymptotics for relaxation times, including the prefactor of the exponential term (Eyring–Kramers laws) are known, for instance, if the stochastic differential equation involves a gradient drift term and homogeneous noise. One approach that has been very successful in proving Eyring–Kramers laws, developed by Bovier, Eckhoff, Gayrard and Klein around 2005, relies on potential theory.

I will describe Eyring–Kramers laws for some parabolic stochastic PDEs such as the Allen–Cahn equation on the torus. In dimension 1, an Arrhenius law was obtained in the 1980s by Faris and Jona-Lasinio, using a large-deviation principle. The potential-theoretic approach allows us to compute the prefactor, which turns out to involve a Fredholm determinant. In dimensions 2 and 3, the equation needs to be renormalized, which turns the Fredholm determinant into a Carleman–Fredholm determinant.

The talk is based on the following articles:

1) with Barbara Gentz (Bielefeld), Sharp estimates for metastable lifetimes in parabolic SPDEs: Kramers' law and beyond, Electronic J. Probability 18 (24):1–58 (2013)

2) with Giacomo Di Gesù (Vienna) and Hendrik Weber (Warwick): An Eyring–Kramers law for the stochastic Allen–Cahn equation in dimension two, Electronic J. Probability 22 (41):1–27 (2017)

Zdzisław Brzeźniak (University of York)

Stochastic Landau–Lifshitz–Bloch equation

We prove the existence of solutions stochastic Landau–Lifshitz–Bloch equation in d-dimensional domains for $d \leq 3$. We prove the pathwise uniqueness when $d \leq 2$. We prove the existence of an invariant measure for $d \leq 2$ and of stationary solutions for $d \leq 3$.

The essential ingredient of the proof is the weak Feller property as even in bounded domains the classical Feller property seems to fail.

My talk is based on joint works with Ben Goldys (Sydney) and Kim Ngan Le (UNSW).

Fabienne Castell (Aix–Marseille Université)

Multiresolution analysis on graphs using random forests and Markov process intertwining

Joint work with: Luca Avena, Alexandre Gaudillière, and Clothilde Mélot

We propose a new method for performing multiscale analysis of functions defined on the vertices of a finite connected weighted graph. Our approach relies on a random spanning forest to downsample the set of vertices, and on approximate solutions of Markov intertwining relation to provide a subgraph structure and a filterbank which is a basis of the set of functions. Our construction involves two parameters q and q'. The first one controls the mean number of kept vertices in the downsampling, while the second one is a tuning parameter between space localization and frequency localization. We provide an explicit reconstruction formula, bounds on the reconstruction operator norm and on the error in the intertwining relation. These bounds lead to recommend a way to choose the parameters q and q'. We illustrate the method by simulations.

Maximilian Engel (Imperial College London)

Quasi-stationary dynamics and bifurcations of random dynamical systems

We look at Markov processes that induce a random dynamical system evolving in a domain with forbidden states constituting a trap. The process is said to be killed when it hits the trap and it is assumed that this happens almost surely. We investigate the behavior of the process before being killed, asking what happens when one conditions the process to survive for a long time.

The topic goes back to the pioneering work by Yaglom in 1947 but in recent years new ideas have been developed. We discuss concepts like quasi-stationary and quasi-ergodic distributions, calling the associated random dynamics quasi-stationary or quasi-ergodic if such distributions exist. Given their existence, we can define average Lyapunov exponents and the Dichotomy spectrum of the random dynamical system with killing and describe the bifurcation behavior of typical examples of stochastic bifurcation theory within this environment. The underlying philosophy is to exhibit the local character of random bifurcations for stochastic differential equations which are usually hidden in the global analysis. We further relate these concepts to dynamical systems with holes.

Erika Hausenblas (Montanuniversität Leoben)

The nonlinear Schrödinger Equation driven by Levy noise

In the talk we first give a short overview about deterministic results. Then, the main results about the existence and uniqueness of the Schrödinger Equation are presented.

Martin Kolb (Universität Paderborn)

Local asymptotics for the area of random walk excursions

We prove a local limit theorem for the area of the positive excursion of random walks with zero mean and finite variance complementing previous results.

Raphael Kruse (TU Berlin)

On randomized one-step methods for non-autonomous (stochastic) differential equations with time-irregular coefficients

In this talk we discuss the numerical solution of non-autonomous (stochastic) differential equations whose coefficient functions are non-smooth. For instance, we are interested in ODEs of Carathéodory type, whose vector field is allowed to be discontinuous or contains a weak singularity with respect to the temporal parameter.

Already in the deterministic case it is well-known that standard numerical solvers only converge very slowly to the exact solution or might even be divergent, if they only depend on finitely many point evaluations of the coefficient function. We therefore focus on certain *randomization* strategies for numerical one-step methods, which often offer several advantages. In particular, we will see that in the case of Carathéodory type ODEs the order of convergence of a randomized solver is increased by 0.5 compared to its deterministic counter-part under the same regularity assumptions on the coefficient functions.

In a further application we consider a randomized Milstein method for stochastic differential equations. In this case it will turn out, that we achieve the same order of convergence as the standard Milstein method but under significantly relaxed smoothness assumption on the drift coefficient function.

This talk is based on a joint work [KW2017, KW2017b] with Yue Wu.

[KW2017] R. Kruse and Y. Wu. Error analysis of randomized Runge–Kutta methods for differential equations with time-irregular coefficients. *Comput. Methods Appl. Math.*, 2017 (to appear).

[KW2017b] R. Kruse and Y. Wu. A randomized Milstein method for stochastic differential equations with non-differentiable drift coefficients. *Preprint*, 2017.

Felix Lindner (TU Kaiserslautern)

Strong approximation of constrained stochastic dynamics

In this talk I present a strong approximation result for a class of stochastic mechanical systems with nonlinear holonomic constraints. Such systems are described by higher-index stochastic differential–algebraic equations, involving an implicitly given Lagrange multiplier process. The explicit representation of the Lagrange multiplier leads to an underlying stochastic ordinary differential equation, whose coefficients are in general not globally Lipschitz continuous and of super-linear growth. Strong convergence is established for a half-explicit drift-truncated Euler scheme which fulfills the constraint exactly. Concrete examples for the considered systems are bead-rod chain models used in molecular dynamics as well as spatially discretized models for the dynamics of inextensible fibers in turbulent flows as occurring, e.g, in the spunbond production process of nonwoven textiles.

The talk is based on joint work with Holger Stroot, ITWM Kaiserslautern.

Grégory Maillard (Aix–Marseille Université)

Parabolic Anderson model in a dynamic random environment: random conductances

We consider a version of the Parabolic Anderson model where the underlying random walk is driven by random conductances and investigate the effect on the Lyapunov exponents. We will show that the annealed Lyapunov exponents are controlled by pockets where the conductances are close to the value that maximises the growth in the homogeneous setting (constant conductances). In contrast, the quenched Lyapunov exponent is controlled by a mixture of pockets where the conductances are nearly constant.

(joint work with Frank den Hollander and Dirk Erhard)

Alexandra Neamtu (Friedrich-Schiller-Universität Jena)

On the Oseledets splitting for infinite-dimensional random dynamical systems

We consider an abstract nonautonomous evolution equation driven by a fractional Brownian motion with Hurst index $H \in (1/2, 1)$. We prove that its solution operator generates a random dynamical system and investigate its long-time behavior. To this aim, we verify the assumptions of the Oseledets multiplicative ergodic theorem (MET) for Banach space-valued cocycles established by Z. Lian and K. Lu (2010). Moreover, using techniques from the center manifold theory, we provide a constructive method of the Oseledets subspaces. This talk is based on a joint work with K. Lu and B. Schmalfuß.

Julian Newman (Universität Bielefeld)

Topological conjugacy of iterated random homeomorphisms of the circle

The natural notion of "isomorphism" for continuous self-maps of a metric space is topological conjugacy. Now a random self-map of a metric space can be studied from a "dynamical" point of view by considering i.i.d. iterations; working from this point of view, the definition of "topological conjugacy" can be extended to the context of a random pair of self-maps of a metric space. Under some reasonable assumptions, we will provide a complete characterisation of topological conjugacy for random orientation-preserving homeomorphisms of the circle.

Martin Redmann (WIAS Berlin)

A regression method to solve parabolic rough PDEs

In this talk, we discuss the numerical approximation of parabolic rough PDEs driven by a path of a fractional Brownian motion. By using the Feynman–Kac formula, the solution can be represented as the expected value of a functional of the corresponding hybrid Stratonovich-rough differential equation. A time-discretisation of this equation and a Monte Carlo regression in the spatial variable lead to an approximation of the solution to the rough PDE. We provide several numerical experiments to illustrate the performance of our method.

Sebastian Riedel (TU Berlin)

Random dynamical systems and rough paths

Rough paths theory (in the sense of Lyons) is a pathwise calculus which can be used to solve stochastic differential equations. We prove that rough differential equations naturally induce random dynamical systems provided the driving path satisfies a version of the cocycle property. This gives rise to the study of new random dynamical systems which are not necessarily Markovian. In particular, we can study convergence to equilibrium, the existence of attractors and stable manifolds for SDEs driven by a fractional Brownian motion.

Joint work with I. Bailleul (Rennes) and M. Scheutzow (Berlin).

Andreas Rößler (Universität zu Lübeck)

Numerical methods for stochastic differential–algebraic equations — some recent developments

Joint work with D. Küpper and A. Kværnø

In the present talk, stochastic ordinary differential equations (SODEs) driven by some Brownian motion and coupled with some algebraic constraints are considered. Such equations are called stochastic differential–algebraic equations (SDAEs) and are applied for, e.g., circuit simulation. The aim of the talk is to give a brief introduction to this topic and to figure out some of the problems that typically arise for the implementation of the corresponding numerical solution methods. The focus of this talk is on so-called stiffly accurate stochastic Runge–Kutta methods that do not involve any pseudo-inverses or projectors for the numerical solution of the problem. Further, we discus the important aspect of mean-square stability and present some A-stable schemes that can be applied to SODEs as well.

Sotirios Sabanis (University of Edinburgh)

Recursive estimators and MCMC algorithms

Some recent advances on recursive estimators with discontinuity in the parameters will be discussed (joint work with Huy N. Chau, Chaman Kumar, Miklos Rasonyi). If time permits, a note on numerics for SDEs and MCMC algorithms will be highlighted.

Michael Scheutzow (TU Berlin)

Minimal random attractors

It is well-known that a random attractor of a random dynamical system which attracts all compact sets is unique while this is not true for a random point attractor (which attracts all deterministic points). We show that if a random point attractor exists then there is always a smallest such point attractor (no matter whether the attraction is in the pullback sense or in probability). We also provide generalizations to other families of attracted sets and provide an example showing that a smallest forward attractor may not exist. This is joint work with Hans Crauel (Frankfurt).

David Siska (University of Edinburgh)

L^p-estimates and regularity for SPDEs with monotone semi-linear term

Semilinear stochastic partial differential equations are considered on a bounded domain $\mathscr{D}$. The semilinear term may have arbitrary polynomial growth as long as it is continuous and it is monotone except perhaps near the origin. The main result here is an L^p-estimates for such equations. The L^p-estimates then provide a way of obtaining higher regularity. It is shown that the solution is continuous in time with values in the Sobolev space $H^2(\mathscr{D}')$ and L^2-integrable with values in $H^3(\mathscr{D}')$, for any $\mathscr{D}'$ that is open and compactly contained in $\mathscr{D}$. Analogous results are also obtained in weighted Sobolev spaces on the whole $\mathscr{D}$.

Konstantinos Spiliopoulos (Boston University)

Hypoelliptic multiscale Langevin diffusions: large deviations, invariant measures and small mass asymptotics

We consider a general class of non-gradient hypoelliptic Langevin diffusions and study two related questions. The first one is large deviations for hypoelliptic multiscale diffusions. The second one is small mass asymptotics of the invariant measure corresponding to hypoelliptic Langevin operators and of related hypoelliptic Poisson equations. The invariant measure corresponding to the hypoelliptic problem and appropriate hypoelliptic Poisson equations enter the large deviations rate function due to the multiscale effects. Based on the small mass asymptotics we derive that the large deviations behavior of the multiscale hypoelliptic diffusion is consistent with the large deviations behavior of its overdamped counterpart. Additionally, we rigorously obtain an asymptotic expansion of the solution to the related density of the invariant measure and to hypoelliptic Poisson equations with respect to the mass parameter, characterizing the order of convergence. The proof of convergence of invariant measures is of independent interest, as it involves an improvement of the hypocoercivity result for the kinetic Fokker–Planck equation. We do not restrict attention to gradient drifts and our proof provides explicit information on the dependence of the bounds of interest in terms of the mass parameter.

Isabell Vorkastner (TU Berlin)

Connectedness of random attractors

Topological properties of random attractors are essential to understand the asymptotic behavior of random dynamical systems. We consider time-continuous random dynamical systems on a connected space where compact sets converge uniformly to the random attractor and analyze whether this attractor is connected.

These attractors are of interest since they are known to be connected in the deterministic setup. In contrast to attractors of time-discrete dynamical systems or point attractors which do not need to be connected. We show that random attractors, which attract compact sets almost surely, are connected. Moreover, we present an example where compact sets converge to the attractor in probability and the attractor is not connected.

This is joint work with M. Scheutzow.

Christian Wiesel (Universität Bielefeld)

Noise-induced synchronization in a ring of weakly coupled oscillators

We study the exchange of energy and the evolution of phase differences in a ring of weakly coupled oscillators. Applying an averaging principle for diffusion processes allows us to show that weak multiplicative noise can lead to amplifcation of some of the system's eigenmodes. In particular, this enables us to characterize those coupling topologies which lead to noise-induced synchronization.

Mathematical Colloquium of the Faculty of Mathematics

Friday, 16 June 2017, 17:15 h, Room V2-210/216

Vojkan Jakšić (McGill University)

Time and entropy

This talk concerns mathematical theory of the so-called Fluctuation Relation (FR) and Fluctuation Theorem (FT) in context of dynamical systems relevant to physics. The FR refers to a certain universal identity linked to statistics of entropy production generated by a reversal operation and FT to the related mathematical large deviations result. The discovery of FR goes back to numerical experiments and Evans, Cohen and Morris (1993) and theoretical works of Evans and Searles (1994), Gallavotti and Cohen (1995). These discoveries generated an enormous body of numerical, theoretical and experimental works which have fundamentally altered our understanding of non-equilibrium physics, with applications extending to chemistry and biology. In this talk I will introduce modern theory of FR and FT and comment on a current research program on this topic.

Tea will be served from 16:30 h on in V3-201.

Workshop dinner (Friday, 16 June 2017, starting at 19:30 h)

Wernings Weinstube

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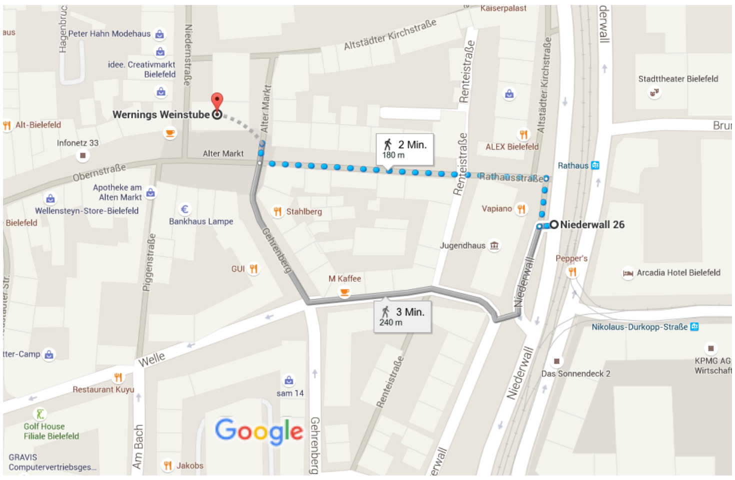

Please note: For the dinner, prior registration is required.

Registered participants

Daniel Altemeier	Universität Bielefeld
Ellen Baake	Universität Bielefeld
Ľubomír Baňas	Universität Bielefeld
Alina Baumheier	Universität Bielefeld
Christian Bayer	WIAS Berlin
Nils Berglund	Université d'Orléans
Wolf-Jürgen Beyn	Universität Bielefeld
Zdzisław Brzeźniak	University of York
Fabienne Castell	Aix–Marseille Université
Jamil Chaker	Universität Bielefeld
Lars Diening	Universität Bielefeld
Christian Döding	Universität Bielefeld
Maximilian Engel	Imperial College London
Barbara Gentz	Universität Bielefeld
Erika Hausenblas	Montanuniversität Leoben
Jakob Herrenbrück	Universität Paderborn
Elena Isaak	Universität Bielefeld
Vojkan Jakšić	McGill University
Diana Kämpfe	Universität Bielefeld
Moritz Kaßmann	Universität Bielefeld
Martin Kolb	Universität Paderborn
Timo Krause	Universität Bielefeld
Raphael Kruse	TU Berlin
Vanessa Lewecke	Universität Bielefeld
Felix Lindner	TU Kaiserslautern
Chengcheng Ling	Universität Bielefeld
Grégory Maillard	Aix–Marseille Université
Alexandra Neamtu	Friedrich-Schiller-Universität Jena
Julian Newman	Universität Bielefeld
Amine Oussama	Universitetet i Oslo
Martin Redmann	WIAS Berlin
Sebastian Riedel	TU Berlin
Andreas Rößler	Universität zu Lübeck
Sotirios Sabanis	University of Edinburgh
Julia Schäpers	Universität Bielefeld
André Schenke	Universität Bielefeld
Michael Scheutzow	TU Berlin
David Siska	University of Edinburgh
Konstantinos Spiliopoulos	Boston University
Christian Vieth	Universität Bielefeld

Isabell Vorkastner	TU Berlin
Christian Wiesel	Universität Bielefeld
André Wilke	Universität Bielefeld
Lukas Wresch	Universität Bielefeld
Rongchan Zhu	Universität Bielefeld
Xiangchan Zhu	Universität Bielefeld

(as of 2 June 2017)

For your notes