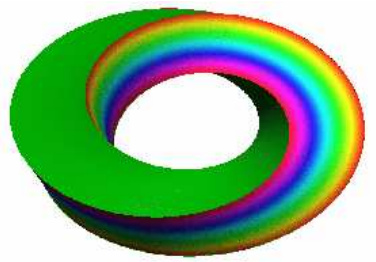
Universität Bielefeld

Fakultät für Mathematik



Fourth Workshop on Random Dynamical Systems

3 – 5 November 2010

Department of Mathematics
University of Bielefeld
Lecture Hall H10

This workshop is part of the conference program of the DFG-funded CRC 701
Spectral Structures and Topological Methods in Mathematics
at the University of Bielefeld

Organizers: Nils Berglund and Barbara Gentz

`http://www.math.uni-bielefeld.de/~gentz/pages/WS10/RDS10/RDS10.html`

Programme

Wednesday, 3 November 2010

9:00 – 9:30	*Registration and coffee*
9:30 – 9:35	*Welcome*
9:35 – 10:20	**Volker Betz** (University of Warwick) Breaking a chain of particles: the role of mass and inter-particle potential
10:35 – 11:00	*Coffee break*
11:00 – 11:45	**Christian Kuehn** (Max Planck Institute for the Physics of Complex Systems, Dresden) Hunting French ducks in a noisy environment
12:00 – 14:30	*Lunch break*
14:30 – 15:15	**Marco Romito** (Università di Firenze) Uniqueness and blow-up for dissipative stochastic PDE
15:30 – 16:00	*Coffee break*
16:00 – 16:45	**Hendrik Weber** (University of Warwick) Rough Burgers-like equations with multiplicative noise
17:00 – 17:45	**Dirk Blömker** (Universität Augsburg) The impact of degenerate noise on a change of stability

Thursday, 4 November 2010

9:00 – 9:30	*Registration and coffee*
9:30 – 10:15	**Michael Röckner** (Universität Bielefeld) *The global random attractor for a class of stochastic porous media equations*
10:30 – 11:00	*Coffee break*
11:00 – 11:45	**Patrick Dondl** (Universität Bonn) Pinning and depinning of interfaces in random media
12:00 – 14:30	*Lunch break*
14:30 – 15:15	**Julian Tugaut** (Universität Bielefeld) Convergence of a self-stabilizing process
15:30 – 16:00	*Coffee break*
16:00 – 16:45	**Gioia Carinci** (Università degli Studi de L'Aquila) Random perturbations of critical equilibria: application to hysteresis and conduction
17:00 – 17:45	**Grant Lythe** (University of Leeds) Kinks and nucleation in a stochastic PDE
19:30 –	*Joint dinner in the city centre* fabrikart, Münzstraße 5, 33602 Bielefeld, phone +49 521 525 1899 (Please note: For the dinner, prior registration is required.)

Friday, 5 November 2010

9:00 – 9:30	*Coffee*
9:30 – 10:15	**Peter Kloeden** (Goethe-Universität Frankfurt a. M.) The numerical stability of stochastic ordinary differential equations with additive noise
10:30 – 11:00	*Coffee break*
11:00 – 11:45	**Peter Reimann** (Universität Bielefeld) Paradoxical noise-effects far from thermal equilibrium
12:00 – 14:00	*Lunch*
14:00 – 14:45	**Reinhard Höpfner** (Johannes-Gutenberg-Universität Mainz) To which extent are semimartingale models adequate for the membrane potential in a neuron?
15:00 – 15:45	— *cancelled* — **Peter Imkeller** (Humboldt-Universität zu Berlin) Meta-stability of some reaction-diffusion equations with Lévy noise
16:00	*Closing of the workshop and coffee*

Abstracts

Volker Betz (University of Warwick)

Breaking a chain of particles: the role of mass and inter-particle potential

We investigate a chain of Brownian particles, coupled by a compactly supported pair potential. The energy is minimal when the particles have a given preferred mutual distance. The chain is pulled at one end until it breaks, and we are interested in the location of the break point, depending on the scaling of the (small) speed of pulling and the (small) noise level. In particular, we investigate how sensitive this scaling is with respect to some modelling assumptions, namely the overdamped versus Langevin dynamics, and the details of the inter-particle potential.

This is joint work with Michael Allman and Martin Hairer.

Dirk Blömker (Universität Augsburg)

The impact of degenerate noise on a change of stability

Using the natural separation of time-scales near a change of stability the dynamics of complicated stochastic partial differential equations can be reduced to amplitude equations, given by ordinary differential equations describing the evolution of the dominant modes (or pattern). Noise acting not directly on the dominant dynamics can nevertheless have an impact due to nonlinear interaction. An interesting observation is the stabilization (i.e., the shift of the bifurcation) due to additive noise.

In the talk, we consider semilinear parabolic PDEs like the stochastic Swift–Hohenberg or the Burger's equation, and present both, numerical and analytical results.

Joint work with Wael W. Mohammed (Augsburg).

Gioia Carinci (Università degli Studi de L'Aquila)

Random perturbations of critical equilibria: application to hysteresis and conduction

My talk is about the influence of random perturbations in dynamical systems in critical regimes.
I start with a problem concerning a Langevin equation for a particle moving in a periodic potential. In absence of stochastic noise there are both confined and unbounded orbits. I show that, with the inclusion of the noise, only the confined orbits survive.
Then I present an hysteresis problem for the mean-field Ising model. Dynamical hysteresis is a phenomenon which arises in ferromagnetic systems below the critical temperature as a response to adiabatic variations of the external magnetic field. I show that for frequencies of the magnetic field oscillations of order $N^{-\frac{2}{3}}$, N the size of the system, the "critical" hysteresis loop becomes random.

Patrick Dondl (Universität Bonn)

Pinning and depinning of interfaces in random media

We consider the evolution of an interface, modeled by a parabolic equation, in a random environment. The randomness is given by a distribution of smooth obstacles of random strength. To provide a barrier for the moving interface, we construct a positive, steady state supersolution. This construction depends on the existence, after rescaling, of a Lipschitz hypersurface separating the domain into a top and a bottom part, consisting of boxes that contain at least one obstacle of sufficient strength. We prove this percolation result.

Furthermore, we examine the question of existence of a solution propagating with positive velocity in a random field with non-bounded random obstacle strength.

This work shows the emergence of a rate independent hysteresis in systems subject to a viscous microscopic evolution law through the interaction with a random environment.

Joint work with N. Dirr (Bath University) and M. Scheutzow (TU Berlin).

Reinhard Höpfner (Johannes-Gutenberg-Universität Mainz)

To which extent are semimartingale models adequate for the membrane potential in a neuron?

We consider sets of data where the membrane potential in a pyramidal neuron (belonging to a cortical slice observed in vitro) is recorded under different experimental conditions. Assuming that the membrane potential between successive spikes can be modelled as a time homogeneous diffusion process, nonparametric estimates for diffusion coefficient and drift make appear relevant classes of models.

Considering power variations (either for fixed powers $p = 2$ or $p = 4$ as a function of the step size, or for fixed step size as a function of the power p) and relying on recent results of Jacod and Ait-Sahalia AS 2009, we then ask the question to which extent a (continuous) semimartingale model is in fact adequate for the membrane potential between successive spikes. Based on our data, we obtain the following conclusion: (i) in non-or very rarely spiking cases the membrane potential behaves as an semimartingale, in some cases with jumps; (ii) once the neuron is sufficiently frequently spiking, an Ito semimartingale modelization is no longer adequate for the membrane potential between successive spikes, even if interspike intervals are relatively long.

Peter Imkeller (Humboldt-Universität zu Berlin)

Meta-stability of some reaction-diffusion equations with Lévy noise

Motivated by studies from climate physics, we investigate a class of reaction-diffusion equations with additive α-stable Lévy noise, stochastic perturbations of the Chafee-Infante equation. We study exit and transition between meta-stable states of their solutions. Due to the heavy-tail nature of an α-stable noise component, the results differ strongly from the well known case of purely Gaussian perturbations. As opposed to the Gaussian picture, where a potential diffusion has to climb to the nearest saddle of the underlying potential landscape for which it takes a time depending exponentially on the noise amplitude, α-stable exits occur as big enough jumps of the noise process and are polynomial in its amplitude. In the scale of these transition times, the diffusion reduces to a pure jump Markov chain between the stable equilibria of the Chafee-Infante equation.

Peter Kloeden (Goethe-Universität Frankfurt a. M.)

The numerical stability of stochastic ordinary differential equations with additive noise

An asymptotic stability analysis of numerical methods used for simulating stochastic differential equations with additive noise is presented. The initial part of the talk is intended to provide a clear definition and discussion of stability concepts for additive noise equation derived from the principles of stability analysis based on the theory of random dynamical systems. The numerical stability analysis presented in the second part of the talk is based on the semi-linear test equation $\mathrm{dX(t)} = (\mathrm{AX(t)} + \mathrm{f(X(t))})\mathrm{dt} + \sigma \mathrm{dW(t)}$, the drift of which satisfies a contractive one-sided Lipschitz condition, such that the test equation allows for a pathwise stable stationary solution. The θ-Maruyama method as well as linear implicit and two exponential Euler schemes are analysed for this class of test equations in terms of the existence of a pathwise stable stationary solution. The latter methods are specifically developed for semi-linear problems as they arise from spatial approximations of stochastic partial differential equations.

Joint work with Evelyn Buckwar and Martin Riedler.

Christian Kuehn (Max Planck Institute for the Physics of Complex Systems, Dresden)

Hunting French ducks in a noisy environment

In this talk we present a study on the influence of noise on oscillations in multiple time scale systems. We introduce the normal form of a folded node singularity in three dimensions with two slow variables and one fast variable. In this context, we encounter canard "duck") orbits that produce local oscillations. The key analytical technique are the analysis of two variational equations and concentration estimates for meta-stable sample paths near invariant manifolds. The results are expected to form a main building block to understand mixed-mode oscillations that appear in many different applications.

Joint work with: Nils Berglund and Barbara Gentz

Grant Lythe (University of Leeds)
Kinks and nucleation in a stochastic PDE

Kinks are localised coherent structures that are a striking feature of noisy, nonlinear, spatially-extended systems in one space dimension with local bistability. At late times, a steady-state density is dynamically maintained: kinks are nucleated in pairs, diffuse and annihilate on collision. Long-term averages can be calculated using the transfer-integral method, giving exact results that can be compared with large-scale numerical solutions of the SPDE. From numerical solutions of an SPDE with a symmetric double-well structure, and with a reduced model in which kinks are treated as point particles, quantities such as the mean number of particles per unit length can be calculated. The dynamics of nucleation events, producing new kink-antikink pairs, are considered as a function of the length of a domain with periodic boundary conditions. The quantities of interest are the width of a newly-nucleated region and the rate of nucleation events.

Peter Reimann (Universität Bielefeld)
Paradoxical noise-effects far from thermal equilibrium

Various new transport phenomena in periodic systems far from thermal equilibrium are addressed. In a first part, symmetry conditions for extracting useful work out of random fluctuations (ratchet effect) are discussed by way of simple theoretical models and illustrated for various experimental systems.
A second part is devoted to the phenomenon of negative mobility in structured micro-fluidic systems. Finally, a new concept is demonstrated of how periodic potentials can be exploited for sorting small objects which only differ by their chirality.

Michael Röckner (Universität Bielefeld)
The global random attractor for a class of stochastic porous media equations

We prove new L^2-estimates and regularity results for generalized porous media equations "shifted by" a function-valued Wiener path. To include Wiener paths with merely first spatial (weak) derivates we introduce the notion of "ζ-monotonicity" for the non-linear function in the equation. As a consequence we prove that stochastic porous media equations have global random attractors. In addition, we show that (in particular for the classical stochastic porous media equation) this attractor consists of a random point.

Marco Romito (Università di Firenze)

Uniqueness and blow-up for dissipative stochastic PDE

Motivated by the recent work on the stochastic Navier–Stokes equations, we analyse an oversimplified model which nevertheless preserves most of the main characteristics of the model. We show well-posedness of the deterministic problem for mildly strong nonlinearity and blow-up with positive probability for strong nonlinearity.

Julian Tugaut (Universität Bielefeld)

Convergence of a self-stabilizing process

A self-stabilizing process corresponds to a particle in a mean-field random dynamical system whose the dimension is infinity. Benachour, Roynette and Vallois proved the weak convergence of this kind of processes. Cattiaux, Guillin and Malrieu extended this result by adding the gradient of a convex potential in the drift term. Carrillo, McCann and Villani proved a similar result in non-convex case by assuming the center of mass is fixed.

By using the non-uniqueness of the stationary measures and the free-energy functionnal, I will prove the convergence under a condition on the moments of the initial value. Moreover, a simple hypothesis under which the process converges weakly towards the unique asymmetrical stationary measure whose the first moment is positive will be provided.

Hendrik Weber (University of Warwick)

Rough Burgers-like equations with multiplicative noise

In this talk we discuss the concept of solutions to one-dimensional Burgers like equations of the type

$$du = \Delta u + g(u)\partial_x u + \theta(u)dW(t)$$

where dW denotes space–time white noise.

Due to the roughness of the noise classical solution concepts are not well defined in this situation. We define weak solutions using rough integration theory to define the spatial integral against a test function. The solution one obtains in this way are subject to a choice similar to the choice between Itô and Stratonovich integration for SDE.

We obtain global existence and uniqueness for our solution once this choice is made. Furthermore, we argue that a particular choice is natural as the solution one obtains is the limit of smoother approximations.

This is a joint work with Martin Hairer.

Registered participants

Daniel Altemeier	(Universität Bielefeld)
Nils Berglund	(MAPMO–CNRS, Orléans)
Volker Betz	(University of Warwick)
Lucian Beznea	(Romanian Academy, Bucharest)
Wolf-Jürgen Beyn	(Universität Bielefeld)
Philippe Blanchard	(Universität Bielefeld)
Dirk Blömker	(Universität Augsburg)
Gioia Carinci	(Università degli Studi de L'Aquila)
Viktorija Carkova	(University of Latvia)
Jevgenijs Carkovs	(University of Latvia)
Patrick Dondl	(Universität Bonn)
Barbara Gentz	(Universität Bielefeld)
Reinhard Höpfner	(Johannes-Gutenberg-Universität Mainz)
Thorsten Hüls	(Universität Bielefeld)
Peter Imkeller	(Humboldt-Universität zu Berlin)
Konrad Klepel	(Universität Augsburg)
Peter Kloeden	(Goethe-Universität Frankfurt a.M.)
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Wei Liu	(Universität Bielefeld)
Grant Lythe	(University of Leeds)
Denny Otten	(Universität Bielefeld)
Kilian Raschel	(Universität Bielefeld)
Peter Reimann	(Universität Bielefeld)
Michael Röckner	(Universität Bielefeld)
Marco Romito	(Università di Firenze)
Jens Rottmann-Matthes	(Universität Bielefeld)
David Siska	(Universität Bielefeld)
Julian Tugaut	(Universität Bielefeld)
Hendrik Weber	(University of Warwick)
Heinz Weisshaupt	(Universität Freiburg)
Sven Wiesinger	(Universität Bielefeld)
Meihua Yang	(Huazhong University of Science and Technology Wuhan, China)
Rongchan Zhu	(Universität Bielefeld)
Xiangchan Zhu	(Universität Bielefeld)

(as of 5 November 2010)