Sixth Workshop on Random Dynamical Systems

31 October – 2 November 2013

Department of Mathematics
University of Bielefeld
Room V2–210/216

This workshop is part of the conference programme of the DFG-funded CRC 701
Spectral Structures and Topological Methods in Mathematics
at the Faculty of Mathematics at the University of Bielefeld

Organizer: Barbara Gentz

http://www.math.uni-bielefeld.de/~gentz/pages/WS13/RDS13/RDS13.htm

Programme

Thursday, 31 October 2013

8:30 – 9:00	*Registration and coffee in V3–201*
9:00 – 9:05	**Friedrich Götze** (Speaker of the CRC 701) *Opening of the workshop (in V2–210/216)*
9:05 – 9:50	**Sabine Jansen** (Ruhr-Universität Bochum) Metastability for continuum interacting particle systems
10:00 – 10:45	**Annika Lang** (Chalmers University of Technology, Gothenburg) Isotropic Gaussian random fields on the sphere
10:50 – 11:15	*Coffee break in V3–201*
11:15 – 12:00	**Reinhard Höpfner** (Johannes-Gutenberg-Universität Mainz) Ergodicity for stochastic Hodgkin–Huxley models driven by an Ornstein–Uhlenbeck process
12:10 – 12:35	**Eva Lang** (TU Berlin) Traveling waves in stochastic neural field equations
12:40 – 14:00	*Lunch break*
14:00 – 14:25	**Sébastien Dutercq** (Université d'Orléans) Kramers' law for Markovian jump processes with symmetries
14:35 – 15:20	**Nils Berglund** (Université d'Orléans) From random Poincaré maps to stochastic mixed-mode oscillation patterns
15:30 – 15:55	**Daniel Altemeier** (Universität Bielefeld) A concentration result for linear stochastic differential equations with delay
16:00 – 16:25	*Coffee break in V3–201*
16:25 – 17:10	**Benjamin Gess** (Universität Bielefeld) Finite time extinction for stochastic sign fast diffusion and self-organized criticality
17:20 – 18:05	**Gilles Wainrib** (Université Paris 13) Complexity of random neural networks

All talks will take place in V2–210/216.

Friday, 1 November 2013

8:45 – 9:00	*Coffee in V3–201*
9:00 – 9:45	**Dirk Blömker** (Universität Augsburg) Accuracy and stability of the continuous-time 3DVAR-filter for 2D Navier–Stokes equation
9:55 – 10:40	**Mykhaylo Evstigneev** (Universität Bielefeld) Stochastic modeling of nanoscale friction phenomena
10:45 – 11:10	*Coffee break in V3–201*
11:10 – 11:55	**Christian Kuehn** (TU Wien) Two perspectives on travelling waves and stochasticity
12:05 – 12:50	**Alexei Novikov** (Pennsylvania State University, State College, PA) Exit times of diffusions with incompressible drift
12:55 – 14:10	*Lunch break* (Catered lunch in V3–201)
14:10 – 14:55	**L'ubomír Baňas** (Universität Bielefeld) Numerical approximation of micromagnetic models with temperature effects
15:05 – 15:50	**Evelyn Buckwar** (Johannes-Kepler-Universität Linz) Mean-square stability of stochastic linear two-step methods for SDEs
15:55 – 16:20	*Coffee break in V3–201*
16:20 – 17:05	**Ilya Pavlyukevich** (Friedrich-Schiller-Universität Jena) Limiting behaviour of finite and infinite dimensional stochastic systems driven by stable Lévy processes
17:15 – 18:00	**Michael Högele** (Universität Potsdam) The first exit problem from the vicinity of the global attractor for dynamical systems perturbed by regularly varying Levy noise
19:00 –	*Joint dinner in the city centre at Wernings Weinstube* (Alter Markt 1, 33602 Bielefeld, ☎ +49 521 136 51 51) Please note: For the dinner, prior registration is required.

All talks will take place in V2–210/216.

Saturday, 2 November 2013

8:45 – 9:00	*Coffee in V3–201*
9:00 – 9:45	**Martin Sauer** (TU Berlin) Analysis and approximation of spatially extended neural systems subject to noise
9:55 – 10:40	**Eric Luçon** (Université René Descartes, Paris 5) The influence of the disorder in the Kuramoto model
10:45 – 11:10	*Coffee break in V3–201*
11:10 – 11:35	**Adam Andersson**(Chalmers University of Technology, Gothenburg) A new approach to weak convergence of SPDEs
11:45 – 12:30	**Raphael Kruse** (ETH Zürich) A multilevel Monte-Carlo theorem for stable numerical methods
12:40 – 13:25	**Peter Kloeden** (Goethe-Universität Frankfurt a.M.) Random ordinary differential equations and their numerical approximation

All talks will take place in V2–210/216.

Abstracts

Daniel Altemeier (Universität Bielefeld)

A concentration result for linear stochastic differential equations with delay

Physical or biological systems are often described by ordinary differential equations. While modeling unresolved degrees of freedom by noise yields (ordinary) stochastic differential equations, more realistic models might require to consider a time-delayed feedback. Thus we are led to study stochastic delay differential equations (SDDEs), which are much harder to analyze due to their non-Markovian nature.

After a brief review of basic concepts and techniques for analyzing stability of deterministic linear delay differential equations (DDEs), we will discuss the effect of Gaussian white noise on DDEs in the stable regime. Employing a suitable representation of the solution to the SDDE, we obtain a good control on the variance process. As the main result we show how to derive concentration results for the sample paths of the SDDE from a classical concentration result for Gaussian processes.

Adam Andersson (Chalmers University of Technology, Gothenburg)

A new approach to weak convergence of SPDEs

Weak convergence of numerical approximations for non-linear SPDEs has previously been proved by a use of the Itô formula and the Kolmogorov equation. In this talk I will present a new method to prove weak convergence for semilinear equations with additive noise, not relying neither on the Itô formula nor on the Kolmogorov equation. We linearize the weak error and obtain a remainder term with high order of convergence. For the linearized term we use the fact that the gradient of the test function, evaluated at the solution of the SPDE, is a Malliavin smooth random variable. This allows us to estimate this term by taking the supremum over a bounded subset of random variables from the Malliavin space. After some analysis a use of the Gronwall Lemma is possible. The order of weak convergence, is as expected, twice that of strong convergence.

The novelty of this result, except for being a new method of proof, is that it allows for test functions with polynomial growth, meaning that we have proved convergence of any moment.

This is joint work with Raphael Kruse (ETH) and Stig Larsson (Chalmers).

L'ubomír Baňas (Universität Bielefeld)

Numerical approximation of micromagnetic models with temperature effects

The Landau–Lifshitz–Gilbert (LLG) equation is a nonlinear partial differential equation that describes magnetization dynamics in ferromagnetic materials. The classical LLG model holds for constant temperatures that are sufficiently far from the Curie temperature. In recent applications, such as the thermally-assisted magnetic recording, it is essential to account for temperature effects in the model. We review two most common approaches currently used for the modelling of thermally activated dynamics in ferromagnetic materials. The first approach includes thermal fluctuations via a random term and leads to a stochastic PDE, the so-called Stochastic-LLG equation. The second approach includes temperature effects via an additional term in the LLG equation and leads to a deterministic model. We propose structure preserving finite element based numerical approximations for the respective equations and show some numerical experiments to demonstrate interesting features of the two models.

Nils Berglund (Université d'Orléans)

From random Poincaré maps to stochastic mixed-mode oscillation patterns

We are interetested in the effect of Gaussian white noise on slow-fast dynamical systems with one fast and two slow variables. In absence of noise, these systems can display mixed-mode oscillations, which are oscillation patterns in which small-amplitude and large-amplitude oscillations (or spikes) alternate. The effect of weak noise can be quantified by analysing the continuous-space, discrete-time Markov chain describing the returns of sample paths to a Poincaré surface of section. An unexpected result of the analysis is that in certain cases, noise may increase the number of small-amplitude oscillations between consecutive spikes.

Joint work with Barbara Gentz (Bielefeld) and Christian Kuehn (Vienna).

Dirk Blömker (Universität Augsburg)

Accuracy and stability of the continuous-time 3DVAR-filter for 2D Navier–Stokes equation

We consider a noisy observer for an unknown solution of a deterministic model. The observer is a stochastic model similar to the original one. It arises as a limit of frequent observations in filtering, where noisy observations of the low Fourier modes together with knowledge of the deterministic model are used to track the unknown solution.

As a toy example we discuss the simple 3DVAR-filter and the deterministic 2D-Navier Stokes equation in this talk. We establish stability and accuracy of the filter by studying the stochastic PDE describing the continuous time observer.

For stability solutions of the observer starting from different initial conditions converge exponentially fast towards each other. This also implies that the random attractor of the observer is at most a single random point.

For accuracy any solution of the observer converges exponentially fast to a small neighbourhood of the unknown solution. In terms of random attractors the whole attractor is close to unknown solution.

Joint work with Andrew Stuart, Kody Law, and Konstantinos Zygalakis.

Evelyn Buckwar (Johannes-Kepler-Universität Linz)

Mean-square stability of stochastic linear two-step methods for SDEs

In this talk we present a linear stability analysis of two-step Maruyama-type methods, for simplicity of notation applied to a scalar stochastic differential equation (SDE), although the analysis also works for a system of SDEs. The main issue is that we obtain a stability matrix, which reflects the asymptotic mean-square stability behaviour of the approximations, and that can be analysed by deterministic methods. We also provide stability plots.

This is joint work with Thorsten Sickenberger.

Sébastien Dutercq (Université d'Orléans)

Kramers' law for Markovian jump processes with symmetries

Consider a reversible Markovian jump process in which the sites are stable or metastable points. We will recall results on the asymmetric case, where the exponentially small eigenvalues of the generator and their relations to hittings times are well known. Then we will discuss an approach to the case where the generator is invariant under a group of symmetries, which is based on representation theory of finite groups. This approach yields precise expressions for the eigenvalues' prefactors, thereby extending Kramers' law to jump processes with symmetries.

Joint work with Nils Berglund (Orléans).

Mykhaylo Evstigneev (Universität Bielefeld)

Stochastic modeling of nanoscale friction phenomena

Direct molecular dynamical simulations of a nanoscale object involve numerical integration of the equations of motion for all its atoms. This method is exact, but is characterized by severe time-scale and size limitations. Stochastic modeling is a useful alternative approach, where one focuses on just a few relevant degrees of freedom, whereas the remaining irrelevant ones are incorporated by introduction of dissipation and noise into the so-called Langevin equations of motion for the relevant coordinates.

In this talk, we will discuss the derivation of the Langevin equations for a nanoscale system in contact with a heat bath. In contrast to the majority of the previous derivations assuming linear system-bath coupling, we will consider a general case where the corresponding interaction potential has an arbitrary functional form, but is weak in comparison to the coupling between the bath particles. Then, the ideas of stochastic modeling will be applied to atomic friction phenomena, namely, the stick-slip motion of an atomic force microscope cantilever, friction aging, and nanoparticle manipulation.

Benjamin Gess (Universität Bielefeld)

Finite time extinction for stochastic sign fast diffusion and self-organized criticality

We will first shortly review the informal derivation of a continuum limit for the Bak–Tang–Wiesenfeld model of self-organized criticality. This will lead to the stochastic sign fast diffusion equation. A key property of models exhibiting self-organized criticality is the relaxation of supercritical states into critical ones in finite time. However, it has remained an open question for several years, whether the continuum limit – the stochastic sign fast diffusion – satisfies this relaxation in finite time. We will present a proof of this.

Michael Högele (Universität Potsdam)

The first exit problem from the vicinity of the global attractor for dynamical systems perturbed by regularly varying Levy noise

The asymptotic first exit time problem for Lévy diffusions at small noise intensity may provide qualitative insight in some paleoclimatic phenomena. A prominent example are α-stable diffusions which arise in the statistical anaysis of time series of the last glacial period, both in the physical and in the mathematical literature.

In this talk we will focus on a result about the first exit problem of regularly varying Lévy diffusions to leave the vicinity of a global attractor under some uniform ergodicity condition for the zero noise deterministic system.

This is joint work with I. Pavlyukevich, FSU Jena.

Reinhard Höpfner (Johannes-Gutenberg-Universität Mainz)

Ergodicity for stochastic Hodgkin–Huxley models driven by an Ornstein–Uhlenbeck process

The classical deterministic Hodgkin–Huxley model (a 4d dynamical system) exhibits a broad range of qualitatively quite different regimes for its solution, with desired periodic behavior (regularly spiking neuron) only in special situations.

Our stochastic Hodgkin–Huxley (HH) system is a 5d SDE containing the classical Hodgkin–Huxley equations together with –as 5th component– an autonomous stochastic process $(\xi_t)_{t\geq 0}$ whose *increments* $d\xi_t$ replace the deterministic input terms $c\,dt$ (constant input) or $S(t)dt$ (periodic input) in the classical Hodgkin–Huxley equation for the voltage. We take ξ as a mean-reverting Ornstein-Uhlenbeck type diffusion with time dependent drift, mean-reverting towards $S(t)$ at time t. Thus a given deterministic periodic signal $t \to S(t)$ is coded in the semigroup of the stochastic Hodgkin–Huxley system.

Control arguments show that with positive probability, our stochastic HH system can mimick the behaviour of almost every deterministic HH (having input $\widetilde{c}\,dt$ constant or $\widetilde{S}(t)dt$ periodic) over time intervals of arbitrary fixed length.

As a 5d SDE, our stochastic HH system driven by 1d Brownian motion is strongly degenerate. Nevertheless, we can prove that it admits continuous Lebesgue densities with respect to 5d Lebesgue measure. This combines calculation of Lie brackets at equilibrium points of classical deterministic HH with control arguments thanks to analyticity of all coefficients. As a consequence, we obtain that weak Hoermander condition holds at all points of the 5d state space.

A Lyapunov function argument shows that some compact K in 5d state space is visited infinitely often by the stochastic HH system in the long run. This allows for Nummelin splitting with respect to specific balls in K, and establishes that there are (at most finitely many) disjoint Harris sets for the stochastic HH system, each Harris set corresponding to an extremal invariant measure.

Joint work with Eva Löcherbach, Université Cergy-Pontoise, and Michele Thieullen, Université Paris VI.

Sabine Jansen (Ruhr-Universität Bochum)

Metastability for continuum interacting particle systems

This talk reports on joint work in progress with Frank den Hollander. We consider a system of point particles in a finite box in $\mathbb{R}^2$ that interact via finite-range attractive pair potential, and evolve according to a Markov process that has the grand-canonical Gibbs measure as a reversible measure. The chemical potential is such that the system favors a packed box, but has a nucleation barrier to overcome in order to go from an empty box to a filled box. We are interested in the nucleation time in the limit as the temperature tends to zero.

We use the potential-theoretic approach to metastability. The results should extend earlier work for lattice systems; the main difficulty lies in understanding the energy landscape of the continuum particle system, a problem of intrinsic interest in analysis.

Peter Kloeden (Goethe-Universität Frankfurt a.M.)

Random ordinary differential equations and their numerical approximation

Random ordinary differential equations (RODEs) are pathwise ordinary differential equations that contain a stochastic process in their vector field functions. They have been used for many years in a wide range of applications, but have been very much overshadowed by stochastic ordinary differential equations (SODEs). The stochastic process could be a fractional Brownian motion, but when it is a diffusuion process there is a close connection between RODEs and SODEs through the Doss-Sussmann transformation and its generalisations, which relate a RODE and an SODE with the same (transformed) solutions. RODEs play an important role in the theory of random dynamical systems and random attractors.

Classical numerical schemes such as Runge-Kutta schemes can be used for RODEs but do not achieve their usual high order since the vector field does not inherit enough smoothness in time from the driving process. It will be shown how, nevertheless, Taylor expansions of the solutions of RODES can be obtained when the stochastic process has Hölder continuous sample paths and then used to derive pathwise convergent numerical schemes of arbitrarily high order. Modifications for stiff RODEs will also be discussed and illustrated with an application in epidemiology.

References

[1] Y. Asai, E. Herrmann and P.E. Kloeden: *Stable integration of stiff random ordinary differential equations,* Stoch. Anal. Appl. 31 (2013), 293–313

[2] A. Jentzen and P. E. Kloeden: *Pathwise Taylor schemes for random ordinary differential equation,* BIT 49 (2009), 113–140

[3] P.E. Kloeden and A. Jentzen: *Pathwise convergent higher order numerical schemes for random ordinary differential equations,* Proc. R. Soc. A 463 (2007), 2929–2944

Christian Kuehn (TU Wien)

Two perspectives on travelling waves and stochasticity

In this talk, I shall outline two challenging directions at the interface of probability, functional analysis and dynamical systems. The first topic is the existence of travelling waves for Nagumo-type equations driven by nonlocal operators arising as macroscopic limits of jump-diffusion processes; this is joint work with Franz Achleitner (Vienna). The second topic are early-warning signs for travelling waves in Fisher–KPP–Nagumo-type SPDEs with a focus on numerical results near wave propagation failure.

Raphael Kruse (ETH Zürich)

A multilevel Monte-Carlo theorem for stable numerical methods

If numerical schemes are used for the discretization of stochastic (partial) differential equations they often need to be combined with Monte Carlo methods. To this regard multilevel Monte-Carlo methods (MLMC) have gained a lot of attention in recent years since they are easily implemented and noticeably reduce the computational complexity compared to standard Monte Carlo methods.

In this talk we connect the MLMC theorem to the classical notion of consistency and stability for numerical methods. This more abstract result is then used to derive the computational complexity of the so called antithetic MLMC sampler, which was recently proposed by M. Giles and L. Szpruch.

Annika Lang (Chalmers University of Technology, Gothenburg)

Isotropic Gaussian random fields on the sphere

Random fields play an important role, e.g., as ingredients for stochastic and random partial differential equations as well as randomly deformed surfaces. In this talk isotropic Gaussian random fields on the sphere are introduced. The approximation with convergence rates and the sample regularity of these fields are discussed, which are of special importance for numerical algorithms. Finally, the random fields are used as increments of Q-Wiener processes, the driving noise of the considered stochastic partial differential equations. Convergence rates of spectral approximations of the stochastic heat equation are related to the regularity of the covariance of the driving noise.

Eva Lang (TU Berlin)

Traveling waves in stochastic neural field equations

The propagation of activity in a neural network consisting of a large number of neurons with nonlocal interactions is in the continuum limit described by an integro–differential equation, the so called neural field equation. Assuming that there are two stable states of the system, "active" and "inactive", it can be shown that there exists a traveling wave solution connecting the two states. Taking extrinsic random influences into account, we realise the one-dimensional neural field equation as a function valued stochastic evolution equation and use a geometric approach to analyse the stability and the speed of the traveling wave.

Eric Luçon (Université René Descartes, Paris 5)

The influence of the disorder in the Kuramoto model

I will discuss some results concerning the large population asymptotics of the Kuramoto model (results also valid for more general models of neurons in interaction, e.g. FitzHugh–Nagumo oscillators). The Kuramoto model has been the first (and simplest) model describing synchronization phenomena. It concerns a population of interacting oscillators in the presence of a random quenched disorder. The way this disorder is chosen strongly influences the behavior of the system in large population and large time. This influence can be captured by computing the quenched law of large numbers and fluctuations of the empirical measure of the oscillators.

Part of my talk concerns joint works with C. Poquet and G. Giacomin and with W. Stannat.

Alexei Novikov (Pennsylvania State University, State College, PA)

Exit times of diffusions with incompressible drift

Consider a Brownian particle in a prescribed time-intependent incompressible flow in a bounded domain. We investigate how the strength of the flow and its geometric properties affect the expected exit time of the particle. The two main questions we analyze in this talk are as follows. 1) Incompressible flows are known to enhance mixing in many contexts, but do they also always decrease the exit time? We prove that the answer is no, unless the domain is a disk. 2) Suppose the flow is cellular with amplitude A, and the domain is of size L. What could be said about the exit time when both L and A are large? We prove that there are two characteristic regimes: a) if $L \ll A^4$, then the exit time from the entire domain is compatible with the exit time from a single flow cell, and it can be determined from the Freidlin–Wentzell theory; b) if $L \gg A^4$, then the problem 'homogenizes' and the exit time is determined by the effective diffusivity of cellular flows.

Ilya Pavlyukevich (Friedrich-Schiller-Universität Jena)

Limiting behaviour of finite and infinite dimensional stochastic systems driven by stable Lévy processes

We consider one dimensional Langevin equations with linear and non-linear friction as well as multi-(infinitely-) dimensional linear Langevin equations driven by stable Lévy processes. We study their asymptotic behaviour in the limit of large friction with the focus on the convergence in the Skorokhod M_1-topology.

Martin Sauer (TU Berlin)

Analysis and approximation of spatially extended neural systems subject to noise

We present recent results on the analysis and approximation of neural systems subject to channel noise fluctuations, capable of describing realisitic features in the nervous system. We study the widely used finite difference approximations for any neuronal model with quasi-monotone coefficients, a natural assumption in the biophysical context. In particular, we obtain strong convergence relying only on a one-sided Lipschitz condition for the nonlinearity, with an implicit rate depending on the regularity of the exact solution. This can be made explicit if the variational solution has more than its canonical spatial regularity. Applications of our results include spatially extended stochastic FitzHugh–Nagumo equations and the estimation of the probability of faithful signal transmission of a pulse along the axon using a new, robust estimator.

This is a joint work with Wilhelm Stannat and funded by the BMBF (grant no. 01GQ1001B), Project A12.

Gilles Wainrib (Université Paris 13)

Complexity of random neural networks

We investigate the explosion of complexity arising near the phase transition to chaos in random neural networks. We show that the mean number of equilibria undergoes a sharp transition from one equilibrium to a very large number scaling exponentially with the dimension on the system. Near criticality, we compute the exponential rate of divergence, called topological complexity. Strikingly, we show that it behaves exactly as the maximal Lyapunov exponent, suggesting a deep and underexplored link between topological and dynamical complexity.

Registered participants

Daniel Altemeier	Universität Bielefeld
Adam Andersson	Chalmers University of Technology, Gothenburg
L'ubomír Baňas	Universität Bielefeld
Nils Berglund	MAPMO–CNRS, Université d'Orléans
Wolf-Jürgen Beyn	Universität Bielefeld
Dirk Blömker	Universität Augsburg
Evelyn Buckwar	Johannes-Kepler-Universität Linz
Sébastien Dutercq	Université d'Orléans
Mykhaylo Evstigneev	Universität Bielefeld
Benjamin Gess	Universität Bielefeld
Barbara Gentz	Universität Bielefeld
Friedrich Götze	Universität Bielefeld
Michael Högele	Universität Potsdam
Reinhard Höpfner	Johannes-Gutenberg-Universität Mainz
Simon Holbach	Johannes-Gutenberg-Universität Mainz
Sander van der Hoog	Universität Bielefeld
Elena Isaak	Universität Bielefeld
Sabine Jansen	Ruhr-Universität Bochum
Diana Kämpfe	Universität Bielefeld
Peter Kloeden	Goethe-Universität Frankfurt a.M.
Christian Kuehn	TU Wien
Raphael Kruse	ETH Zürich
Annika Lang	Chalmers University of Technology, Gothenburg
Eva Lang	TU Berlin
Eric Luçon	Université René Descartes, Paris 5
Katharina von der Lühe	Universität Bielefeld
Nora Müller	Universität Bielefeld
Ziad Muslimani	Florida State University, Tallahassee, FL / Heinrich-Heine-Universität Düsseldorf
Alexei Novikov	Pennsylvania State University, State College, PA
Ilya Pavlyukevich	Friedrich-Schiller-Universität Jena
Peter Reimann	Universität Bielefeld
Michael Röckner	Universität Bielefeld
Martin Sauer	TU Berlin
Gilles Wainrib	Université Paris 13
Christian Wiesel	Universität Bielefeld

(as of 25 October 2013)