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## Some notation

- $\mathbb{R}_+ \equiv (0, +\infty)$
- $\text{esup}$  – the essential supremum
- $\text{einf}$  – the essential infimum
- $f_+ \equiv \frac{1}{2}(|f| + f)$  - the positive part
- $f_- \equiv \frac{1}{2}(|f| - f)$  - the negative part
- $[f]_a^b \equiv f(b) - f(a)$
- $\log_+ x \equiv (\log x)_+$
- $\simeq$  “comparable to”; namely,  $f(x) \simeq g(x)$  if there exists a constant  $C > 0$  such that  $C^{-1}g(x) \leq f(x) \leq Cg(x)$  for all  $x$  from a specified domain.
- $B_r(x)$  – a ball in  $\mathbb{R}^n$ , that is,  $B_r(x) = \{y \in \mathbb{R}^n : |x - y| < r\}$
- $B_r \equiv B_r(0) = \{y \in \mathbb{R}^n : |y| < r\}$ .
- $\omega_n$  – the area of the unit  $(n - 1)$ -sphere in  $\mathbb{R}^n$ .
- $1_A$  – the indicator function of a set  $A$ , that is,  $1_A(x) = 1$  if  $x \in A$  and  $1_A(x) = 0$  otherwise.
- $\Subset$  “compact inclusion”;  $A \Subset B$  means that the closure  $\bar{A}$  of the set  $A$  is compact and  $\bar{A} \subset B$ .
- $\mathcal{H}$  – a Hilbert space
- $\rightharpoonup$  the sign of the weak convergence (in a Hilbert space)
- $M$  – a Riemannian manifold
- $x \rightarrow \infty$  – a sequence of points on a manifold eventually leaving any compact set.
- $g$  – the Riemannian metric on  $M$
- $\mu$  – a reference measure on  $M$
- $\Delta_\mu$  – the weighted Laplace operator on  $M$
- $\mathcal{L}$  – the Dirichlet Laplace operator on  $M$
- $L^p(M, \mu)$  – the Lebesgue function space
- $\|\cdot\|_p \equiv \|\cdot\|_{L^p}$
- $u \leq v \pmod{W_0^1}$  means that  $u \leq v + w$  where  $w \in W_0^1$ .
- $d$  – the geodesic distance on  $M$
- $B(x, r)$  – a geodesic ball on  $M$  with respect to the geodesic distance  $d(x, y)$ .

*Conventions.*

- Summation is assumed over repeated indices. For example,

$$\xi_i x^i = \sum_{i=1}^n \xi_i x^i, \quad a^{ij} u_j = \sum_{j=1}^n a^{ij} u_j, \quad g_{ij} v^i v^j = \sum_{i,j=1}^n g_{ij} v^i v^j,$$

etc.

- Letters  $c, C, c', C'$ , etc denote positive constants (depending on specified parameters) whose value may change at each occurrence.
- positive  $\equiv$  strictly positive, negative  $\equiv$  strictly negative, decreasing  $\equiv$  non-increasing, increasing  $\equiv$  non-decreasing

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