Michael Butler
From abelian groups to strings and bands

Henning Krause

Universität Bielefeld

Sherbrooke, October 4, 2013

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Michael Charles Richard Butler (1929 – 2012)

Short Curriculum Vitae
M. C. R. Butler
May 18, 2005

Full Name	Michael Charles Richard BUTLER.
Date, place of birth	6 January 1929, Melbourne, Australia.
Nationality	British.
Work address	Department of Mathematical Sciences, University of Liverpool, Liverpool, L69 7ZL, UK.
Home address	37 Sydenham Avenue, Liverpool, L17 3AU, UK.
Home telephone, email	44-(0)151-734-0034, mcrb@liv.ac.uk.
Academic Degrees	B.Sc.(1949),M.A.(1951),Ph.D.(1955), all from the University of Melbourne, Australia.
Status	Retired. Honorary Senior Fellow in Pure Mathematics, University of Liverpool, since 1997.
Employment record	Lecturer, Senior Lecturer and Reader in Pure Mathematics, University of Liverpool, 1957-96; Head of Department, 1983-88.
Research interests	Homological algebra, abelian groups, representation theory of finite-dimensional algebras and orders.

M.C.R. Butler: London Mathematical Society obituary

MICHAEL BUTLER

Published online 05 April 2013

Dr Michael Charles Richard Butler, who was elected a member of the London Mathematical Society on 15 December 1955, died on 18 December 2012, aged 83.

Peter Giblin writes (with advice from Mary Rees and Claus Ringel): Michael and his wife Sheila Brenner, who died in 2002, were active members of the mathematics departments of the University of Liverpool from the time of their appointments in 1957, when they both moved from the University of London. Until the merger of the two departments (and Statistics) in the 1990s Sheila was in the Department of Applied Mathematics and Michael in the Department of Pure Mathematics; but from the early 1960s, and a joint research leave to Michael's home country of Australia, they worked together on problems in algebra.

Michael's earlier work was devoted to questions in homological algebra. His detailed study of a class of torsion-free groups of finite rank (now called Butler groups) showed the complexity of such groups. His use of representations of posets in order to study abelian groups was very influential as one of the first general reduction techniques. In several papers he described the surprising dichotomy between tame and wild behaviour of module categories. In their joint work Michael and Sheila developed 'tilting theory', now an indispensable tool in algebra and geometry providing a general framework for dealing with equivalences of triangulated categories. Their first major publication on this was in 1980: *Generalizations of the Bernstein-Gelfand-Ponomarev reflection functors*, in the proceedings of the second ICRA (International Conference on Representations of Algebras). From its beginning, Michael was one of the scientific advisors for the ICRA conference series which started in 1974 in Ottawa, Canada, and now is held every second year in different countries. Michael and Sheila's last joint publication was in 2007, five years after Sheila's death. Together, they organized a very successful symposium at the University of Durham in 1985.

Michael was a highly successful Head of the (then) Department of Pure Mathematics in Liverpool in the 1980's: perhaps surprisingly so, given his strong, forthrightly expressed, and even unfashionable, left-wing views, which were also an important part of his partnership with Sheila. But he also had exceptional organisational ability, and was naturally kind, courteous, pragmatic and level headed. Michael formally retired in 1996 but continued active in work and conference attendance until his medical condition prevented it. Michael and Sheila had no children, but Michael, from a large family, has dozens of collateral descendants, and will also be missed by his many friends around the world.

Most important publications

Author Citations for Michael C. R. Butler

Michael C. R. Butler is cited 409 times by 306 authors

in the MR Citation Database

Most Cited Publications

Citations	Publication
111	**MR0876976 (88a:16055)** Butler, M. C. R.; Ringel, Claus Michael Auslander-Reiten sequences with few middle terms and applications to string algebras. *Comm. Algebra* 15 (1987), no. 1-2, 145–179. (Reviewer: Christine Riedtmann) 16A64 (16A35)
89	**MR0607151 (83e:16031)** Brenner, Sheila; Butler, M. C. R. Generalizations of the Bernstein-Gel'fand-Ponomarev reflection functors. *Representation theory, II (Proc. Second Internat. Conf., Carleton Univ., Ottawa, Ont., 1979)*, pp. 103–169, Lecture Notes in Math., 832, *Springer, Berlin-New York*, 1980. (Reviewer: Idun Reiten) 16A64 (16A46)
33	**MR1930968 (2003i:16011)** Brenner, Sheila; Butler, Michael C. R.; King, Alastair D. Periodic algebras which are almost Koszul. *Algebr. Represent. Theory* 5 (2002), no. 4, 331–367. (Reviewer: Peter A. Linnell) 16E05 (16G10 16S37)
32	**MR1670674 (2000f:16013)** Butler, M. C. R.; King, A. D. Minimal resolutions of algebras. *J. Algebra* 212 (1999), no. 1, 323–362. (Reviewer: Dieter Happel) 16E40 (16G20 16G60 16G70)
29	**MR0218446 (36 #1532)** Butler, M. C. R. A class of torsion-free abelian groups of finite rank. *Proc. London Math. Soc. (3)* **15** 1965 680–698. (Reviewer: W. Liebert) 20.30
15	**MR0174593 (30 #4794)** Brenner, Sheila; Butler, M. C. R. Endomorphism rings of vector spaces and torsion free abelian groups. *J. London Math. Soc.* **40** 1965 183–187. (Reviewer: C. W. Curtis) 16.40
14	**MR0230767 (37 #6327)** Butler, M. C. R. Torsion-free modules and diagrams of vector spaces. *Proc. London Math. Soc. (3)* **18** 1968 635–652. (Reviewer: S. B. Conlon) 16.90
13	**MR0225878 (37 #1469)** Butler, M. C. R. On locally free torsion-free rings of finite rank. *J. London Math. Soc.* **43** 1968 297–300. (Reviewer: G. Michler) 20.30 (16.00)
12	**MR0188267 (32 #5706)** Butler, M. C. R.; Horrocks, G. Classes of extensions and resolutions. *Philos. Trans. Roy. Soc. London Ser. A* **254** 1961/1962 155–222. (Reviewer: S. Eilenberg) 18.20

The research fields

- Relative homological algebra
  M.C.R. Butler, G. Horrocks: Classes of extensions and resolutions, 1961.
- Torsion-free abelian groups
  M.C.R. Butler, A class of torsion-free abelian groups of finite rank, 1965.
- Representations of orders and integegral group rings
  M.C.R. Butler: On the classification of local integral representations of finite abelian p-groups, 1974.
- Representations of finite-dimensional algebras
  M.C.R. Butler, C.M. Ringel: Auslander-Reiten sequences with few middle terms and applications to string algebras, 1987.
- Tilting theory
  S. Brenner, M.C.R. Butler: Generalizations of the Bernstein-Gel'fand-Ponomarev reflection functors, 1980.

Relative homological algebra

Butler & Horrocks: Classes of extensions and resolutions, 1961.

- The authors write: *The ideas of relative homological algebra have been formulated for categories of modules by Hochschild (1956), and for abstract categories by Heller (1958) and Buchsbaum (1959). The common feature of these papers is the selection of a class of extensions or, equivalently, a class of monomorphisms and epimorphisms. In Hochschild's paper it is the class of extensions which split over a given subring of the ring of operators.*
- Thus: Relative homological algebra is the study of an abelian category C by looking at certain subfunctors of $\operatorname{Ext}^1_{\mathsf{C}}(-,-)$.
- The center $Z(\mathsf{C})$ of C is introduced as the commutative ring of all endomorphisms $\mathrm{Id}_{\mathsf{C}} \to \mathrm{Id}_{\mathsf{C}}$ of the identity functor.
- Subfunctors of $\operatorname{Ext}_{\mathsf{C}}(-,-)$ arise from $Z(\mathsf{C})$.
- Example: For a ring A, the center $Z(\operatorname{Mod} A)$ is isomorphic to the center $Z(A)$.

Torsion-free abelian groups: Butler groups

Butler: A class of torsion-free abelian groups of finite rank, 1965.

- In abelian group theory, torsion-free groups are notoriously complicated objects. Kaplansky writes (1959):

  In this strange part of the subject anything that can conceivably happen actually does happen.

- Butler writes (1965):

  This paper is concerned with the study of the smallest class of torsion-free abelian groups which (1) contains all groups of rank 1*, and (2) is closed with respect to the formation of finite direct sums, pure subgroups, and torsion-free homomorphic images.*

- These groups are now called Butler groups and allow a description in terms of typesets.

Torsion-free abelian groups: poset representations

Butler: Torsion-free modules and diagrams of vector spaces, 1968.

Brenner & Butler: Endomorphism rings of vector spaces and torsion free abelian groups, 1965

- The description of Butler groups of given typeset T involves the study of poset representations of T.
- The wild behaviour of such categories is studied by realising 'complicated' endomorphism rings.
- Brenner writes: *I am indebted to Dr. M. C. R. Butler for a breakfast-table education in algebra, and for many useful discussions.*

An early understanding of ‘zahm und wild’

An exhibition 1990 in Basel.

A pioneer of the ICRA: first conference in Ottawa, 1974

(PRELIMINARY NOTICE)

Carleton University

Department of Mathematics

cordially invites you to an

International Conference on Representations of Algebras

September 3-7, 1974

(FOLLOWING THE INTERNATIONAL CONGRESS OF MATHEMATICIANS 1974 IN VANCOUVER)

TO DATE, THE FOLLOWING MATHEMATICIANS HAVE ACCEPTED A PRELIMINARY INVITATION:

M. AUSLANDER (BRANDEIS)
R. BRAUER (HARVARD)
S. BRENNER (LIVERPOOL)
M.C.R. BUTLER (LIVERPOOL)
C.W. CURTIS (OREGON)
A.W.M. DRESS(BIELEFELD)
P. GABRIEL (BONN)
H. JACOBINSKI (GOTEBORG)
G.J. JANUSZ (ILLINOIS)
C.U. JENSEN (KOBENHAVN)
M.M. KLEINER (KIEV)

H. KUPISCH (HEIDELBERG)
G. MICHLER (GIESSEN)
L.A. NAZAROVA (KIEV)
C. PROCESI (PISA)
I. REINER (ILLINOIS)
I. REITEN (TRODHEIM)
C.M. RINGEL (TUBINGEN)
K.W. ROGGENKAMP (BIELEFELD)
A.V. ROITER (KIEV)
M. TACHIKAWA (TOKYO)
T. YOSHII (SHIGA)

V. DLAB
FOR THE ORGANIZING COMMITTEE OF ICRA

THOSE WISHING TO OBTAIN THE FIRST NOTICE OF THE CONFERENCE, PLEASE WRITE TO
SECRETARY OF ICRA, DEPARTMENT OF MATHEMATICS, CARLETON UNIVERSITY, OTTAWA, CANADA

Representations of orders and integral group rings

Butler: On the classification of local integral representations of finite abelian p-groups, Proceedings of the International Conference on Representations of Algebras, Ottawa, 1974.

- Butler writes: *Lattices over orders and integer group rings are notoriously complicated objects. A theorem of Dade's shows that 'most' orders have infinite representation type (i.e. infinitely many non-isomorphic indeoomposable lattices).*
- He continues: *This paper develops further a strategy which was shown to work nicely for 2-adic integral representations of the Four Group* $C_2 \times C_2$*. The leading idea is to relate lattice categories to other, better understood categories, primarily, to the categories of vector space representations of quivers or of partially ordered sets.*

Representations of finite-dimensional algebras

INDECOMPOSABLE REPRESENTATIONS OF THE LORENTZ GROUP

I.M. Gel' fand and V.A. Ponomarev

Let L be the Lie algebra of the Lorentz group or, what is the same, of the group $SL(2,C)$. We denote by L_k the Lie algebra of its maximal compact subgroup, that is, of $SU(2)$. Let M_i be the finite-dimensional irreducible L_k-modules (the finite-dimensional representations of L_k). Consider an L-module M. The authors call M a *Harish-Chandra module* if, regarded as L_k-module, it can be written as a sum

$$M = \bigoplus_i M_i$$

of finite-dimensional irreducible L_k-modules M_i. Here, for each M_{i_0}, only finitely many L_k-submodules equivalent to M_{i_0} are supposed to occur in the decomposition of M.

A Harish-Chandra module is called indecomposable if it cannot be decomposed into the direct sum of L-submodules. In this paper the indecomposable Harish-Chandra modules over L are completely described. We find that there are two types of indecomposable Harish-Chandra modules. The modules of the first type are the non-singular Harish-Chandra modules and are defined by the following invariants: an integer $2l_0 (l_0 \geqslant 0)$, a complex number l_1, and an integer n. The first two of these invariants are already known as invariants of the irreducible representations of the Lorentz group (see [2]). The case of non-singular modules has been investigated earlier by Zhelobenko [3] from a somewhat different approach.

The case of singular Harish-Chandra modules is of the greatest interest. The solution of this problem reduces to a non-trivial problem of linear algebra, which is investigated in detail in Chapter 2. The invariants of singular indecomposable modules are, as before, numbers l_0, l_1, $l_0 \geqslant 0$, $2l_0$ integral and $2l_0 - |l_1|$ integral.

However, instead of the one additional invariant n, there are now more invariants. Two types of singular modules are possible: those of the first and those of the second kind.

Singular modules of the first kind are characterized, in addition to the invariants l_0 and l_1, by a sequence of integers of arbitrary length. Singular indecomposable modules of the second kind are characterized by the following collection of invariants: the numbers l_0, l_1 given above, a set of integers $j_1, j_2, \ldots, j_k$, an integer q, and a further arbitrary complex parameter μ. The presence of this parameter is particularly interesting, because it indicates the possibility of deforming an indecomposable module with l_0 and l_1 fixed.

The problems of linear algebra that are used in establishing the facts set out above are of independent interest in that the authors develop and use the apparatus of MacLane's theory of linear relations [4].

COMMUNICATIONS IN ALGEBRA, 15(1&2), 145-179 (1987)

AUSLANDER-REITEN SEQUENCES WITH FEW MIDDLE TERMS AND APPLICATIONS TO STRING ALGEBRAS

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dedicated to Maurice Auslander on his 60 th. birthday.

In the famous paper [AR-III] Auslander and Reiten introduced what now are called Auslander-Reiten sequences, and one consequence has been the definition of several numerical invariants both of individual modules and of artin algebras. Let A be such an algebra. Given an Auslander Reiten sequence

$$0 \longrightarrow X \longrightarrow \bigoplus_{i=1}^{r} Y_i \longrightarrow Z \longrightarrow 0,$$

with all Y_i indecomposable, the number $r = \alpha(Z)$ may be called the <u>number of middle terms</u>, and is defined for all indecomposable non-projective modules. Viewed as a function, α was considered by Auslander and Reiten in [AR-O] where they defined $\alpha(A)$ to be the supremum of $\alpha(Z)$ over all indecomposable non-projective

The functorial filtration method: strings and bands

Butler & Ringel: Auslander-Reiten sequences with few middle terms and applications to string algebras, 1987.

- The authors write: *There are two methods known for obtaining a complete description of the Auslander-Reiten sequences of a string algebra. One method is based on the calculation of the indecomposable modules due to Gelfand-Ponomarev: first, one determines the Auslander-Reiten translate, and then the corresponding Auslander-Reiten sequences. The second method is based on covering theory.*
- They continue: *Our aim is to demonstrate that the Gelfand-Ponomarev technique is well suited to showing that certain maps between indecomposable modules are irreducible, and that, in this way, one obtaines essentially all irreducible maps, and therefore also all Auslander-Reiten sequences.*

Tilting theory: from black magic to tilting functors

GENERALIZATIONS OF THE BERNSTEIN-
GELFAND-PONOMAREV REFLECTION FUNCTORS

Sheila Brenner and M.C.R. Butler

Introduction

Reflection functors were introduced into the representation theory of quivers by Bernstein, Gelfand and Ponomarev in their work on the 4-subspace problem and on Gabriel's Theorem and there have been several generalisations, see [13], [6], [10] and [2]. The aim of this paper is to present a further extension of the concept and to give some applications to quivers with relations (QWR's). A special case of this theory has been developed by Marmaridis [19] and applied to certain QWR's; indeed some of the methods used in his Thesis [18] may also be regarded as applications of these functors, though they are not presented in that way.

Associated with any representation of a quiver is a dimension vector, and the dimension vectors of indecomposable modules are the positive roots of the quadratic form associated to the quiver (see e.g. [6], [10], [15]). Similar results seem to hold for certain QWR's. Some applications of reflection functors involve the study of the transformations of dimension vectors they induce. It turns out that there are applications of our functors which make use of the analogous transformations which we like to think of as a change of basis for a fixed root-system - a tilting of the axes relative to the roots which results in a different subset of roots lying in the positive cone. (An example is considered in some detail in Chapter 4, §2). For this reason, and because the word 'tilt' inflects easily, we call our functors <u>tilting functors</u> or simply <u>tilts</u>.

PhDs at Liverpool (supervised by M.C.R. Butler)

CRUDDIS, Thomas Barry: On a class of torsion free abelian groups, 1964

SHAHZAMANIAN, Mostafa: Representation of Dynkin graphs by abelian *p*-groups, 1979

COELHO, Flávio Ulhoa: Preprojective partitions and Auslander-Reiten quivers for artin algebras, 1990

BURT, William Leighton: Homological theory of bocs representations, 1991

Students at Liverpool (1989/1990)

Sheila Brenner: Henning Krause, Shiping Liu
Michael Butler: Flávio Coelho, William Burt

Drawn into representation theory: postdocs at Liverpool

Some of the postdocs in Liverpool in the 1980/90s

Bill Crawley-Boevey

Mike Prest

Alastair King

LMS Durham Symposium 1985

Representations of Algebras (organisers: M.C.R. Butler, S. Brenner)

ICRA at Tsukuba (1990) and Mexico (1994)

An advocate of the Kiev school

Michael 1997 at a conference in Kiev.

Twenty years of tilting theory (Fraueninsel, 2002)

TWENTY YEARS OF TILTING THEORY

- an Interdisciplinary Symposium -

November 18-22, 2002,
Fraueninsel, Germany

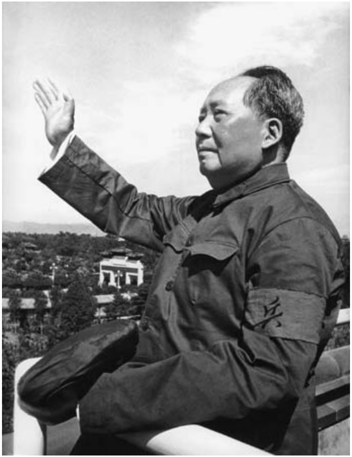
Tilting modules were born about twenty years ago in the context of finite dimensional algebras. Since then, tilting theory has spread in many different directions, and nowadays it plays an important role in various branches of modern algebra, ranging from Lie theory and algebraic geometry to homotopical algebra. The aim of this meeting is to bring together for the first time experts from different fields where tilting is relevant or even of central importance. There will be several lecture series and survey talks on the use of tilting theory in different contexts, as well as a number of additional talks contributed by the participants.

Here is a tentative list of the invited speakers:

M. van den Bergh (University of Limburg)
S. Brenner (University of Liverpool)
T. Brüstle (University of Bielefeld)
M. Butler (University of Liverpool)
S. Donkin (University of London)
K. Erdmann (University of Oxford)
K. Fuller (University of Iowa)
B. Keller (University of Paris VII)
S. König (University of Leicester)
H. Lenzing (University of Paderborn)
O. Mathieu (University of Lyon)
J. Miyachi {Tokyo Gakugei University)
I. Reiten (NTNU Trondheim)
J. Rickard (University of Bristol)
C. M. Ringel (University of Bielefeld)
R. Rouquier (University of Paris VII)
J. Trlifaj (Charles University Prague)

Organizers: Lidia Angeleri Hügel (Munich), Dieter Happel (Chemnitz), Henning Krause (Bielefeld).

A hospitable place: 37 Sydenham Avenue, Liverpool



Shaking hands with Mao

Morning Star

As homophobic as ever

SYMON HILL on how the Conservatives' new-found commitment to gay rights rings rather hollow: p8

Spot-on satire with nobs in

THEATRE: P11

Vote labour

As Britain goes to the polls, the Morning Star says:

Back candidates who back working people

GREEK MASSES DEFEND COUNTRY

Standing up to BAE Systems

NEWS: P4

Shell admits huge Niger Delta leaks

WORLD: P6

Get the facts with our World Cup guide

FOOTBALL: P15

The daily newspaper

From an e-mail to Bielefeld (August 28, 2012)

Your comments on ICRA were interesting. It is exciting that the 'old representation theory' has become so important in applications to areas of applicable mainstream maths, a development which will keep it alive as a subject in its own right (unlike, for example, torsion free abelian group theory!!!!), and maybe lead to solutions of some of the remaining hard problems of pure reprn theory; there is an analogy here with the way 'pure complex function theory' still develops because of its vast applications. Of course it makes life harder for old men like me, but I do really like what is happening.