

## Exercises to Introduction to Stochastic Partial Differential Equations I

Sheet 14

*This exercise sheet is not meant to be handed in and will be discussed in the first tutorial in the next semester. We recommend to use the opportunity to recapitulate the main aspects of the proofs of Theorems 4.2.5 and 4.2.4 in the lecture notes.*

### **Problem 1.**

Let  $V \subset H \subset V^*$  be a Gelfand triple. Assume that  $(X(t))_{t \in [0, T]}$  is a  $V^*$ -valued stochastic process on a probability space  $(\Omega, \mathcal{F}, \mathbb{P})$  such that  $\sup_{t \in [0, T]} \|X(t)\|_H < \infty$   $\mathbb{P}$ -a.s. Suppose that  $X$  is weakly continuous in  $V^*$   $\mathbb{P}$ -a.s. Prove that  $X$  is weakly continuous in  $H$   $\mathbb{P}$ -a.s.

### **Problem 2.**

Summarise the proof of Theorem 4.2.5. Structure the proof in several steps and highlight the main points and stress the difficulties.

### **Problem 3.**

Summarise the proof of Theorem 4.2.4. Structure the proof in several steps and highlight the main points and stress the difficulties.