Summer term 2023
Prof. Dr. Michael Röckner and Sebastian Grube

Exercises to Introduction to Stochastic Partial Differential Equations I

Sheet 9
Total points: 12
Submission before: Friday, 09.06.2023, 12:00 noon

Problem 1 (Prove the details)**.** (2+2+2 Points)

Consider the situation of the proof of Lemma 3.1.4.

(i) Prove (3.1.14). In particular, determine the martingale $(M_R^{(n,m)}(t))_{t\in[0,\infty)}$.

Consider the situation in the proof of Proposition 3.2.1.

(ii) Prove that P-a.e. for all $t \in [0,T]$ and all $n \in \mathbb{N}$

$$|X^{(n)}(t\wedge\gamma^{(n)}(R)) - X(t\wedge\gamma^{(n)}(R))|^2\phi_{t\wedge\gamma^{(n)}(R)}(R) \leqslant |X_0^{(n)} - X_0|^2 e^{-\sup_{n\in\mathbb{N}}|X_0^{(n)}|} + m_R^{(n)}(t),$$

for some continuous local $(\mathcal{F}_t)$-martingales $(m_R^{(n)}(t))_{t\in[0,T]}$ such that $m_R^{(n)}(0) = 0$. Identify $(m_R^{(n)}(t))_{t\in[0,\infty)}$.

Consider the situation in the proof of Theorem 3.1.1.

(iii) Show that $\tau(R)$ (defined below (3.2.3)) is an $(\mathcal{F}_t)$-stopping time for every $R \in [0,\infty)$.

Problem 2. (4 Points)

Consider Section 4.1 in the lecture notes. Prove that $H^* \subset V^*$ continuously and densely, i.e. show that the canonical embedding $i^* : H^* \to V^*$ is a bounded linear operator which is one-to-one and $i^*(H^*) \subset V^*$ is dense.

Problem 3. (2 Points)

Consider Section 4.1 in the lecture notes. Prove that V^* is separable.