Winter term 2023/2024
Lecture: Prof. Dr. Michael Röckner
Exercises: Dr. Sebastian Grube

Exercises to Introduction to Stochastic Partial Differential Equations II

Sheet 6
Total points: 14
Submission before: Friday, 24.11.2023, 12:00 noon

Problem 1 (cf. end of the proof of Lemma 4.3.11)**.** (2 Points)

Consider the situation of Lemma 4.3.9. Give the details on why the following inequality, which appears in the proof of Lemma 4.3.11, holds.

$$E\exp(\lambda t)||X(t,s,x)-X(t,s,y)||_H^2 \leqslant \exp(\lambda s)||x-y||_H^2,\ \forall x\in H, s\in(-\infty,t].$$

How do you conclude from here that η (as constructed in the proof of the same lemma) is actually independent of $x \in H$?

Problem 2 (Conditions on A, B in Chapter 5)**.** (2+2 Points)

Consider the situation in the beginning of Chapter 5 in the lecture notes.

(i) Prove that (H4) from Chapter 4 is equivalent to (H4') if $\beta = 0$.

(ii) Prove Remark 5.1.1 (3), i.e. (H3) and (H4') imply that for all $t \in [0,T], v \in V$

$$\begin{aligned}||B(t,v)||_{L_2(U,H)}^2 \leqslant f(t) + C_0||v||_H^2 + \frac{2(\alpha-1)}{\alpha}f(t)\left(1+||v||_H^\beta\right)\\ + \frac{2}{\alpha}\left[C_0(\alpha-1)\left(1+||v||_H^\beta\right)+1-\frac{\alpha}{2}\theta\right]||v||_V^\alpha \text{ on } \Omega.\end{aligned}$$

Problem 3 (A priori estimate in Chapter 5)**.** (4 Points)

Carry out the details of the proof of Lemma 5.1.4.

Problem 4 (Itô's formula for $||\cdot||^p$)**.** (4 Points)

Provide the details on the first equality in the proof of Lemma 5.1.5.

Hint: Use Lemma 2.4.4.