

# Finding one of $D$ defective elements in some group testing models.

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## Abstract

In contrast to the classical goal of group testing we want to find  $m$  defective elements among  $D$  ( $m \leq D$ ) defective elements. We analyse two different test functions. We give adaptive strategies and lower bounds for the number of tests and show that our strategy is optimal for  $m = 1$ .

Group testing is of interest for many applications like in molecular biology. For an overview of results and applications we refer to the books [1] and [2]. In [3] the authors consider the problem to find one defective element in a finite set, where an element  $i$  is defective with probability  $p_i$ . The case in which all the probabilities are equal occurs already in [4].

In [5] the authors consider the problem of finding at least  $k$  nondefective elements. Their study was motivated by a practical problem of an electronic firm. The production department of the firm required  $10^6$  nondefective electronic chips for their production process. There is a method to test a pool of chips. They buy chips of 99% quality (a chip is defective with probability 0.01) and want to find with a small number of group tests many non-defective elements.

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We want to consider the combinatorial version of this problem. So we want to find  $m$  of  $D$  defective elements. This study was motivated by [6] and [7]. We denote by  $[N] := \{1, 2, \dots, N\}$  the set of elements, by  $\mathcal{D} \subset [N]$  the set of defective elements, by  $D = |\mathcal{D}|$  its cardinality, and by  $[i, j]$  the set of integers  $\{x \in \mathcal{N} : i \leq x \leq j\}$ . Throughout the paper we consider worst case analysis.

The classical group testing problem is to find the unknown subset  $\mathcal{D}$  of all defective elements in  $[N]$ .

For a subset  $\mathcal{S} \subset [N]$  a test  $t_{\mathcal{S}}$  is the function  $t_{\mathcal{S}} : 2^{[N]} \rightarrow \{0, 1\}$  defined by

$$t_{\mathcal{S}}(\mathcal{D}) = \begin{cases} 0 & , \text{ if } |\mathcal{S} \cap \mathcal{D}| = 0 \\ 1 & , \text{ otherwise.} \end{cases} \quad (1)$$

We define search strategies as in [8]. In classical group testing a strategy is called successful, if we can **uniquely determine**  $\mathcal{D}$ . Here we call a strategy successful if we can find one element of  $\mathcal{D}$ . We remind the reader of the concepts of adaptive and nonadaptive strategies.

Strategies are called adaptive if the results of the first  $k - 1$  tests determine the  $k$ th test. Strategies in which we choose all tests independently are called nonadaptive.

Let  $f$  be a function  $f : [0, N] \rightarrow \mathbb{R}^+$ . We define *general group tests with density* as  $t_{\mathcal{S}} : 2^{[N]} \rightarrow \{0, 1\}$ , defined by

$$t_{\mathcal{S}}(\mathcal{D}) = \begin{cases} 0 & , \text{ if } |\mathcal{S} \cap \mathcal{D}| < f(|\mathcal{S}|) \\ 1 & , \text{ if } |\mathcal{S} \cap \mathcal{D}| \geq f(|\mathcal{S}|). \end{cases} \quad (2)$$

In [7] the case  $f(|\mathcal{S}|) = \alpha|\mathcal{S}|$  is considered. The authors assume that a lower bound of the cardinality of  $\mathcal{D}$  is known. **The goal is to find  $m \leq D$  defective elements.**

In **majority group testing** (defined in [9] and more general in [10]) we have two functions  $f_1, f_2 : \{0, 1, \dots, N\} \rightarrow \mathbb{R}^+$  which put weights on the number  $D$  of defective elements and  $f_1(D) \leq f_2(D) \forall D \in [0, 1, \dots, N]$ .

We describe the structure of tests  $t_{\mathcal{S}} : 2^{[N]} \rightarrow \{0, 1, \{0, 1\}\}$  as follows

$$t_{\mathcal{S}}(\mathcal{D}) = \begin{cases} 0 & , \text{ if } |\mathcal{S} \cap \mathcal{D}| < f_1(D) \\ 1 & , \text{ if } |\mathcal{S} \cap \mathcal{D}| \geq f_2(D) \\ \{0, 1\} & , \text{ otherwise} \end{cases} \quad (3)$$

(meaning that the result can be arbitrary 0 or 1).

In [10] it is assumed that the searcher does not know the cardinality of  $\mathcal{D}$  but knows some upper bound. In majority group testing **it is not always**

**possible to find the set  $\mathcal{D}$  of all defective elements** (see [10], [11]). In general, one can **find a family  $\mathbb{F}$  of sets, which contains  $\mathcal{D}$** . This family depends on  $f_1$  and  $f_2$ , on  $\mathcal{D}$ , and on the strategy used. In this case we call a strategy successful, if we can find an  $\mathbb{F}$  with the smallest possible size.

Now we put the ideas of these two models together such that there are two functions  $f_1, f_2 : [0, N] \times [0, N] \rightarrow \mathbb{R}^+$  with  $f_1(D, S) \leq f_2(D, S)$  for all values of  $D$  and  $S$ .

We define a test  $t_S : 2^{[N]} \rightarrow \{0, 1, \{0, 1\}\}$  as follows

$$t_S(\mathcal{D}) = \begin{cases} 0 & , \text{ if } |\mathcal{S} \cap \mathcal{D}| < f_1(D, |\mathcal{S}|) \\ 1 & , \text{ if } |\mathcal{S} \cap \mathcal{D}| \geq f_2(D, |\mathcal{S}|) \\ \{0, 1\} & , \text{ otherwise} \end{cases} \quad (4)$$

(meaning that the result can be arbitrary 0 or 1).

For this test function denote by  $n(N, D, m)$  the minimal number of tests for finding  $m$  defective elements.

The following lower bound for the minimal number of test is a generalization of a theorem in [7]. They give this lower bound for  $f_1(D, |\mathcal{S}|) = f_2(D, |\mathcal{S}|) = \alpha|\mathcal{S}|$ .

**Theorem 1**  $n(N, D, 1) \geq \lceil \log(N - D + 1) \rceil$

**Proof:** Let us assume that we have a successful strategy  $s$  which finds a defective element with  $n = n(N, D, 1)$  tests and  $n < \lceil \log(N - D + 1) \rceil$ .

Depending on the  $n$  test results we have at most  $2^n$  different possible results for a defective element, we denote them by  $\mathcal{E}$ . It holds by assumption that  $|\mathcal{E}| \leq 2^n < N - D + 1$ . Therefore  $|[N] \setminus \mathcal{E}| > D - 1$  and there exists a set  $\mathcal{F} \subset [N] \setminus \mathcal{E}$  with  $|\mathcal{F}| = D$ . Now we consider the case  $\mathcal{D} = \mathcal{F}$ . It is obvious now that strategy  $s$  we cannot find any defective element with  $n$  tests.  $\square$

We consider the following special cases of this test model, where  $f = f_1 = f_2$  and  $D$  is known.

Threshold group testing without gap:  $f(D, |\mathcal{S}|) = u$ . Thus

$$t_S(\mathcal{D}) = \begin{cases} 0 & , \text{ if } |\mathcal{S} \cap \mathcal{D}| < u \\ 1 & , \text{ if } |\mathcal{S} \cap \mathcal{D}| \geq u \end{cases} \quad (5)$$

Group testing with density tests:  $f(D, |\mathcal{S}|) = \alpha|\mathcal{S}|$  for all values. Thus

$$t_S(\mathcal{D}) = \begin{cases} 0 & , \text{ if } |\mathcal{S} \cap \mathcal{D}| < \alpha|\mathcal{S}| \\ 1 & , \text{ if } |\mathcal{S} \cap \mathcal{D}| \geq \alpha|\mathcal{S}|. \end{cases} \quad (6)$$

We consider for all this test functions the adaptive model with the goal of finding one defective element.

In Section 1 we consider the test function (1) (classical case) and give an optimal strategy of finding one out of  $D$  defectives with  $\lceil \log(N - D + 1) \rceil$  tests.

In Section 2 we give strategies for test function (5) (threshold case) and show that the strategy is optimal for  $m = 1$ . Furthermore we combine the strategy with the strategy in [12] of finding  $m$  elements.

In Section 3 we give a strategy for test function (6) and give some remarks on nonadaptive group testing.

## 1 Classical test function

In this section we use the test function (1). We assume that  $D$  ( $0 < D < N$ ) is known. Our goal is to find  $m$  defective elements.

We denote by  $n_{(Cla)}(N, D, m)$  the minimal number of tests (1) of finding  $m$  defective elements.

**Proposition 1**  $n_{(Cla)}(N, D, 1) \leq \lceil \log(N - D + 1) \rceil$

**Proof:** We give a strategy which needs  $\lceil \log(N - D + 1) \rceil$  tests. We know that the set  $\mathcal{S}_0 = \{D, D + 1, \dots, N\}$  contains at least one defective element. Thus we start with the test set  $\mathcal{S}_1 \subset \mathcal{S}_0$  of size  $\lfloor \frac{N-D+1}{2} \rfloor$ . If the test is positive, then at least one defective element is in  $\mathcal{S}_1$ , otherwise at least one defective element is in  $\mathcal{S}_0 \setminus \mathcal{S}_1$ . Therefore depending on the test result we substitute  $\mathcal{S}_0$  by  $\mathcal{S}_1$  or  $\mathcal{S}_0 \setminus \mathcal{S}_1$  and iterate the procedure. With this method we will find one defective element with  $\lceil \log(N - D + 1) \rceil$  tests.  $\square$

Proposition 1 together with Theorem 1 implies the following

**Corollary 1** 1.  $n_{(Cla)}(N, D, 1) = \lceil \log(N - D + 1) \rceil$ ,

2.  $n_{(Cla)}(N, D, m) \leq m \lceil \log(N - D + 1) \rceil$ .

### Remark

1. If we do not know  $D$ , but  $1 \leq D' < D'' < N$  with  $D' \leq D \leq D''$ , then we need  $\lceil \log(N - D' + 1) \rceil$  tests of finding one defective element.

## 2 Threshold test function without gap

We consider now the test function (5). This kind of test was introduced in [11] and called threshold group testing without gap. First we assume that we know  $D$ .

We denote by  $n_{(Thr)}(N, D, u, m)$  the minimal number of tests (5) for finding  $m$  defective elements, if we have  $N$  elements with  $D$  defectives and  $f(D, |\mathcal{S}|) = u$ .

Our first goal is to find one defective element.

**Proposition 2** *If  $D \geq u$  then  $n_{(Thr)}(N, D, u, 1) \leq \lceil \log(N - D + 1) \rceil$ , otherwise it is not possible to find any defective element.*

**Proof:** We give a strategy which needs  $\lceil \log(N - D + 1) \rceil$  tests. The idea of the proof is to partition the set of  $N$  elements into the subsets  $\mathcal{I}_1 = [1, u - 1]$ ,  $\mathcal{I}_2 = [u, N - D + u]$ , and  $\mathcal{I}_3 = [N - D + u + 1, N]$ . In  $\mathcal{I}_2$  there is of course at least one defective, because the union of the two other subsets has cardinality  $D - 1$ . We can find a defective element in  $\mathcal{I}_2$  by the following strategy with  $\lceil \log(N - D + 1) \rceil$  tests.

We start with the test set

$$\mathcal{S}_1 = \{1, \dots, u - 1, u, \dots, (u - 1) + \lceil \frac{m(1)}{2} (N - D + 1) \rceil\},$$

where  $m(1) = 1$ .

Inductively, we set  $m(j) = \begin{cases} 2m(j - 1) - 1 & \text{if } t_{\mathcal{S}_{j-1}}(\mathcal{D}) = 1 \\ 2m(j - 1) + 1 & \text{if } t_{\mathcal{S}_{j-1}}(\mathcal{D}) = 0, \end{cases}$

and  $\mathcal{S}_j = \{1, \dots, u - 1, u, u + 1, \dots, (u - 1) + \lceil \frac{m(j)}{2} (N - D + 1) \rceil\}$ .

After  $\lceil \log(N - D + 1) \rceil$  tests we can find an  $i$  such that  $t_{[1, i]} = 1$ ,  $t_{[1, i-1]} = 0$  because it is clear that  $t_{[1, u-1]} = 0$  and  $t_{[1, N-D+u]} = 1$ . Thus using this strategy we find an defective element at the position  $i$ .  $\square$

From Theorem 1 and Proposition 2 we get the following

**Theorem 2**  $n_{(Thr)}(N, D, u, 1) = \lceil \log(N - D + 1) \rceil$ , if  $D \geq u$ .

The strategy can be generalized for the case of finding  $m$  defective elements.

**Proposition 3** *Let  $D \geq m$  then  $n_{(Thr)}(N, D, u, m) \leq m \lceil \log(N - D + 1) \rceil$ .*

**Proof:** We apply the strategy using in Proposition 2 for finding one defective element. We use the ordered set  $[N]$  and denote by  $\pi_j(i)$  the  $j$ th position before the  $i$ th test. We set  $\pi_j(1) = j$ . In the first round we apply the strategy of Proposition 2 and find the defective element  $d_1$ . Then we set

$$\pi_j(2) = \begin{cases} d_1 & \text{if } j = 1 \\ 1 & \text{if } j = d_1 \\ j & \text{if } j \notin \{1, d_1\} \end{cases}$$

(that means we exchange the elements at the positions  $d_1$  and 1) and apply the same strategy with  $\lceil \log(N - D + 1) \rceil$  tests and find the defective element  $d_2$  for the new set  $\{\pi_1(2), \pi_2(2), \dots, \pi_N(2)\}$ . We exchange now the elements at the positions  $d_2$  and 2 and iterate this procedure and exchange after every round the elements at the positions  $d_j$  and  $j$  until we find the defective element  $d_u$ . From now on we exchange the defective element at the position  $d_j$  with the element at the position  $N - D + 1 + j$ . In total after  $m$  iterations we find  $m$  defectives.  $\square$

**Remark** If we have already found  $u - 1$  defective elements, we can use every classical group testing strategy to find in the set of  $N - u + 1$  unknown elements the remaining  $D - u + 1$  defectives by adding the  $u - 1$  defective elements to every test.

We apply this improvement, if we want to find all defective elements, using the following result of Hwang [12]

$$n_{(Cla)}(N, D, D) \leq \lceil \log \binom{N}{D} \rceil + D - 1,$$

as follows. After  $u - 1$  rounds in the proof of Proposition 3 we use Hwang's strategy for the remaining  $N - u + 1$  elements with  $D - u + 1$  defectives and then we get a total of

$$T(u) = (u - 1)\lceil \log(N - D + 1) \rceil + \lceil \log \binom{N - u + 1}{D - u + 1} \rceil + D - u + 1$$

tests. This gives the following upper bound

**Theorem 3**  $n_{(Thr)}(N, D, u) \leq T(u)$ .

If  $D$  is unknown we can take one test with all elements. Then if the answer is negative we can not find any defective element. If the answer is positive then we know that  $D \geq u$ .

So we are interesting the case we do not know  $D$ , but we have  $u \leq D \leq N$ .

If  $D$  is unknown we denote by  $n_{(Thr)}(N, u, m)$  the minimal number of tests (5) to find  $m$  defective objects in the worst case, if we have  $N$  elements and  $f(|\mathcal{D}|, |\mathcal{S}|) = u$  for all values. In this case we have

**Lemma 1**  $n_{(Thr)}(N, u, m) \leq m \lceil \log(N - u + 1) \rceil$ .

**Proof:** A similar idea works like in the proof of Proposition 3 if  $D$  is unknown. We give a strategy which needs  $m \lceil \log(N - u + 1) \rceil$  tests. We test in  $m$  adaptive rounds and start with the test set  $\mathcal{S}_1 = \{1, \dots, u - 1, u, \dots, (u - 1) + \lceil \frac{m(1)}{2}(N - u + 1) \rceil\}$ , where  $m(1) = 1$ .

We set for  $j \leq \lceil \log(N - u + 1) \rceil$ :

$$m(j) = \begin{cases} 2m(j-1) - 1 & \text{if } t_{\mathcal{S}_{j-1}}(\mathcal{D}) = 1 \\ 2m(j-1) + 1 & \text{if } t_{\mathcal{S}_{j-1}}(\mathcal{D}) = 0, \end{cases}$$

and  $\mathcal{S}_j = \{1, \dots, u - 1, u, \dots, (u - 1) + \lceil \frac{m(j)}{2^j}(N - u + 1) \rceil\}$ .

By testing  $\mathcal{S}_1, \dots, \mathcal{S}_{\lceil \log(N - u + 1) \rceil}$  we find one defective element  $d_1$  with  $\lceil \log(N - u + 1) \rceil$  tests.

We use now instead of the set  $\{1, 2, \dots, N\}$  the set  $\{\pi_1, \pi_2, \dots, \pi_N\}$ , where

$$\pi_j = \begin{cases} d_1 & \text{if } j = 1 \\ 1 & \text{if } j = d_1 \\ j & \text{if } j \notin \{1, d_1\} \end{cases}$$

and continue now as before with  $\lceil \log(N - u + 1) \rceil$  tests and find the defective element  $d_2$  for the new set  $\{\pi_1, \pi_2, \dots, \pi_N\}$ . Then we iterate this procedure until we have found  $u - 1$  defectives. Then we know that in the set  $[u, N]$  are the remaining  $D - u + 1$  defectives objects. These defectives can be found in  $(m - u + 1)$  rounds with  $\lceil \log(N - u + 1) \rceil$  tests.  $\square$

### 3 Density tests

The test model (6) was considered in [7].

Let  $n_{(Den)}(N, D, m, \alpha)$  be the minimal number of tests (6) for finding  $m$  defective elements, if we have  $N$  elements with  $D$  defectives. In [7] the authors obtain the following bounds for  $n_{(Den)}(N, D, m, \alpha)$  assuming  $D \geq \alpha N$

$$\lceil \log N \rceil + \max_{N' \leq 2 \frac{m}{\alpha}} n_{(Den)}(N', m, m, \alpha) \geq n_{(Den)}(N, D, m, \alpha), \quad (7)$$

$$\lceil \log N \rceil \geq n_{(Den)}(N, D, 1, \alpha). \quad (8)$$

In general they show that

$$\log(N - D + 1) \leq n_{(Den)}(N, D, 1, \alpha). \quad (9)$$

The test model (5) give the same result as test model (1) if the size of the test set is smaller than  $\frac{1}{\alpha}$ . In the strategy given in the proof of Proposition 1 the biggest test set  $\mathcal{S}_0$  has cardinality  $\lfloor \frac{N-D+1}{2} \rfloor$ . If in the test model (5)  $|S_0|$  is smaller than  $\frac{1}{\alpha}$  we can apply the strategy and get  $n_{(Den)}(N, D, 1, \alpha) \leq \lceil \log(N - D + 1) \rceil$ . This is the case if  $D \geq N + 1 - \frac{2}{\alpha}$ . Therefore we get

**Proposition 4** *Let  $D \geq N + 1 - \frac{2}{\alpha}$  then  $n_{(Den)}(N, D, 1, \alpha) = \lceil \log(N - D + 1) \rceil$ .*

Now we will improve the result.

We will give a strategy which is optimal for  $D \geq \alpha N$  (it needs  $\lceil \log(N - D + 1) \rceil$  questions).

Let us define

$$s_i = \lceil \frac{2^{n-i} - 1}{1 - \alpha} \rceil$$

where  $i = 1, 2, \dots, n - 1$  and  $s_n = 1$ .

For given  $D$  we choose the maximal  $n$  such that

$$D > \sum_{i=1}^n s_i - 2^n + 1. \quad (10)$$

We consider test sets

$$\mathcal{S}_i = \{a_i + 1, a_i + 2, \dots, a_i + s_i\}$$

for  $i = 1, \dots, n$ , where  $a_1 = 0$  and

$$a_i = \begin{cases} a_{i-1} + s_{i-1} & , \text{ if } t_{\mathcal{S}_{i-1}}(\mathcal{D}) = 0 \\ a_{i-1} & , \text{ if } t_{\mathcal{S}_{i-1}}(\mathcal{D}) = 1. \end{cases} \quad (11)$$

Note that  $|S_i| = s_i$ .

**Lemma 2** *If  $t_{\mathcal{S}_{n-j}}(\mathcal{D}) = 1$  then we can find one defective element after  $n$  tests.*

**Proof:** We proceed by induction on  $j$ . The case  $j = 0$  is obvious. Let us consider also  $j = 1$  (to show the idea of the strategy). We have  $s_{n-1} = \lceil \frac{1}{1-\alpha} \rceil$  and  $t_{S_{n-1}}(\mathcal{D}) = 1$ . It holds

$$s_{n-1} - 2 < \alpha s_{n-1} \leq s_{n-1} - 1.$$

Thus in the set  $S_{n-1}$  we have not more than one nondefective element. If  $t_{S_n}(\mathcal{D}) = 1$  it give us a defective element, otherwise ( $t_{S_n}(\mathcal{D}) = 0$ ) we can take any element from  $S_n \setminus S_{n-1}$ .

We assume that the statement is proved for  $j - 1$ . Let  $t_{S_{n-j}}(\mathcal{D}) = 1$ , then by induction hypothesis we can assume that  $t_{S_{n-i}}(\mathcal{D}) = 0$  for all  $0 \leq i < j$ .

Thus the number of nondefective elements in  $S_{n-j}$  is not bigger than  $2^j - 1$  since  $t_{S_{n-i}}(\mathcal{D}) = 1$  and

$$s_{n-j} - 2^j < \alpha s_{n-j} \leq s_{n-j} - 2^j + 1.$$

On the other hand the number of nondefective elements in  $S_{n-i}$  for all  $0 \leq i < j$  is bigger or equal than  $2^i$  since  $t_{S_{n-i}}(\mathcal{D}) = 0$ .

Thus all elements in  $S_{n-j} \setminus \bigcup_{i < j} S_{n-i}$  are defective.

The set  $S_{n-j} \setminus \bigcup_{i < j} S_{n-i}$  not empty, because it holds

$$1 + \sum_{i=1}^k \lceil \frac{2^i - 1}{1 - \alpha} \rceil < 1 + k + \sum_{i=1}^k \frac{2^i - 1}{1 - \alpha} = 1 + k + \frac{2^{k+1} - k - 2}{1 - \alpha} < \frac{2^{k+1} - 1}{1 - \alpha}$$

for any  $k$  and  $\alpha$  ( $0 < \alpha < 1$ ). □

**Theorem 4** *Let (10) be fulfilled and  $N \leq 2^n + D - 1$  then after  $n$  tests of the strategy above we will find one defective element.*

**Proof:** Consider tests sets defined in (11). If for some  $i$  we have  $t_{S_i}(\mathcal{D}) = 1$  then the theorem follows by Lemma 2. If  $t_{S_i}(\mathcal{D}) = 0$  for all  $i = 1, 2, \dots, n$  then we denote by  $c_i$  the number of nondefective elements in  $S_i$ . The number of defective in  $S_i$  is  $s_i - c_i$ . Thus we have  $s_i - c_i < \alpha s_i$  and hence  $c_i \geq 2^i$ .

In total the number of nondefective elements is not less than  $2^n - 1$  and since  $N - D = 2^n - 1$  we can take any element of  $[N] \setminus \bigcup_{t=1}^n S_t$ . Note if  $N < 2^n + D - 1$  then there is a  $i$  with  $t_{S_i}(\mathcal{D}) = 1$ . □

**Corollary 2** *If  $D \geq \alpha N$  then  $n_{(Den)}(N, D, 1) = \lceil \log(N - D + 1) \rceil$ .*

**Proof:** By (10) it holds

$$D > \sum_{k=0}^{n-1} (\lceil \frac{2^k - 1}{1 - \alpha} \rceil - 2^k).$$

$$n - 1 + \sum_{k=1}^{n-1} (\frac{2^k - 1}{1 - \alpha} - 2^k) = \frac{\alpha}{1 - \alpha} (2^n - n - 1)$$

If we take

$$D > \frac{\alpha}{1 - \alpha} (2^n - n - 1)$$

and

$$N < 2^n + \frac{\alpha}{1 - \alpha} (2^n - n - 1) - 1$$

then

$$\frac{N}{D} < \frac{1 - \alpha}{\alpha} + 1 + \frac{(1 - \alpha)n}{\alpha(2^n - n - 1)}.$$

Thus if  $D \geq \alpha N$  we can apply Theorem 4. □

**Remarks** (The nonadaptive case)

In [6] it is shown that for the test (1) if  $D$  is unknown one needs  $N$  tests of finding one defective element or to claim that there is no defective element. If  $D$  is known, we can test  $N - D$  elements of finding one defective element or we can use a  $(D, 1)$  cover-free code of finding all elements and thus also one.

The nonadaptive model for majority group testing is considered in [9] and [10]. There the goal is to find all defective elements.

The results of [13] for row-weighted cover-free codes can be used to get strategies for test (6) if the number of defectives are known.

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