Palindrome complexity

Jean-Paul Allouche
CNRS, LRI
Université Paris-Sud, Bât. 490
F-91405 Orsay Cedex (France)
allouche@lri.fr

Michael Baake
Institut für Mathematik und Informatik
Universität Greifswald, Jahnstraße 15 a
D-17487 Greifswald (Germany)
mbaake@uni-greifswald.de

Julien Cassaigne
CNRS, IML
Case 907, 163 Avenue de Luminy
F-13288 Marseille Cedex 9 (France)
cassaigne@iml.univ-mrs.fr

David Damanik
Department of Mathematics 253-37
California Institute of Technology
Pasadena, CA 91125 (USA)
damanik@its.caltech.edu

Dedicated to Jean Berstel for his 60th birthday with our very best wishes.

Abstract

We study the *palindrome complexity* of infinite sequences on finite alphabets, i.e., the number of palindromic factors (blocks) of given length occurring in a given sequence. We survey the known results and obtain new results for some sequences, in particular for Rote sequences and for fixed points of primitive morphisms of constant length belonging to "class P" of Hof-Knill-Simon. We also give an upper bound for the palindrome complexity of a sequence in terms of its (block-)complexity.

1 Introduction

The (block-)complexity function of an infinite sequence on a finite alphabet is the number of factors (blocks) of given length occurring in this sequence. This notion was introduced in 1975 by Ehrenfeucht, Lee, and Rozenberg [27]. The complexity function of a sequence measures in some sense how "complicated" the sequence is: the reader is referred to the surveys [3, 29, 30]. Due *inter alia* to its applications to physics [32], another interesting "complexity" for an infinite sequence on a finite alphabet is its *palindrome complexity*, i.e., the number of palindromic factors (blocks) of given length occurring in the sequence. Combinatorial results on palindrome complexity for some sequences or classes of sequences were proved in [24, 4, 26, 10, 19, 21]. We survey known results, and give new ones, proving for example that *the palindrome complexity of Rote sequences is constant and equal to* 2. We prove that *the palindrome complexity function of a sequence that is a fixed point of a primitive morphism of*