My work with Maurice

Idun Reiten
13.11.14
9:30h – 10:30h

① Stable equivalence

Λ f.d. k-algebra $\longmapsto$ $\underline{\mathrm{mod}}\,\Lambda$
(stable category)

$$\underline{\mathrm{Hom}}(A,B) = \mathrm{Hom}_\Lambda(A,B)/P(A,B)$$

$$\begin{tikzcd} & P \arrow[dr] & \\ A \arrow[ur] \arrow[rr] & & B \end{tikzcd}$$

Λ and Λ' are stably equivalent if
$\underline{\mathrm{mod}}\,\Lambda \sim \underline{\mathrm{mod}}\,\Lambda'$

Theorem: Λ is stably equivalent to a hereditary algebra

$\Longleftrightarrow$
(a) Each indecomposable submodule of a projective module is simple or projective
(b) If S is a simple submodule of a projective module, S not projective, then S is a factor of an injective module.

Problem: Do stably equivalent algebras Λ and Λ' have same number of simple nonprojective modules?

Ex: $\mathrm{rad}^2 = 0$ for Λ, then
Λ and $\begin{pmatrix} \Lambda/\mathrm{rad} & 0 \\ \mathrm{rad} & \Lambda/\mathrm{rad} \end{pmatrix}$ are stably equivalent.

Martinez: finite representation type;
Enough to show for selfinjective algebras.

Idun Reiten

13.11.14

9:30-10:30

(2) Almost split sequences (1971 –)

$$\begin{tikzcd} & & & X \text{ indec.} \arrow[dl, dashed] \arrow[d] & \\ 0 \arrow[r] & A \arrow[r] & B \arrow[r] & C \arrow[r] & 0 \end{tikzcd}$$

exact sequence
almost split

$A = D\mathrm{Tr}\, C$

Two existence proofs

1. Functorial one : S simple functor

   Find exact sequence

   $$(-,B) \to (-,C) \to S \to 0$$

2. Guess of relationship between A and C

   $$\tau = D \circ \mathrm{Tr}$$

Teter, Menzin: $k[X,Y]/(X,Y)^2$, groupalgebra $\Omega^2 = D \circ \mathrm{Tr}$

Based on formula:

$$D\,\mathrm{Ext}^1(C, \tau A) \cong \underline{\mathrm{Hom}}(A, C)$$

(3) Commutative algebra (80's)

Idun Reiten
13.11.14
9:30-10:30

Setting: R commutative complete local integrally closed CM domain, Krull dim $= n$, maximal ideal $\mathfrak{m}$

(Assume $T = k[[x_1, \dots, x_n]] \subset R$ such that R is finitely generated free T-module)

$CM(R)$ (= max. CM-mod) R-modules, free as T-modules

- $\exists$ almost split sequences in $CM(R)$ $\iff$ R isolated singularity (i.e. $R_{\mathfrak{p}}$ is regular local for all prime ideals $\neq \mathfrak{m}$)

Assume R is not a hypersurface, Krull dim ≥ 3:

We found two examples of finite representation type. More?

1. Invariant rings

   $G \subset Gl(n, k)$ some $n \geq 3$
   finite

   Then; $k[[x_1, \dots, x_n]]^G$ is of finite representation type

   $\iff n = 3,\ G = \left\langle \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix} \right\rangle$

2. Scroll of type (2,1)

   generally: scroll of type $(n_1, \dots, n_r)$

   Type (2,1): $\left(\begin{array}{cc|c} x_0 & x_1 & y_0 \\ x_1 & x_2 & y_1 \end{array}\right)$. Have ring $\dfrac{k[[x_0, x_1, x_2, y_0, y_1]]}{(x_0x_2 - x_1^2,\ x_1y_1 - x_2y_0,\ x_0y_1 - x_1y_0)}$

   is of finite representation type.

Problem: Any more?

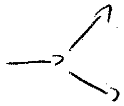
Here: ~~Tr~~ $\Omega \circ (\ )^*$ replaces Tr

$\mathrm{Hom}(-, \omega)$ replaces D

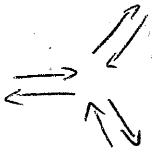
Idun Reiten
13.11.14

· Interplay with finite dimensional algebras

(Alternative proof ~~by~~ of a result of Ringel, Schofield)

Q Dynkin quiver 

⇝ path algebra kQ

⇝ preprojective algebra  $\longleftrightarrow$ $\underline{CM}(R)$

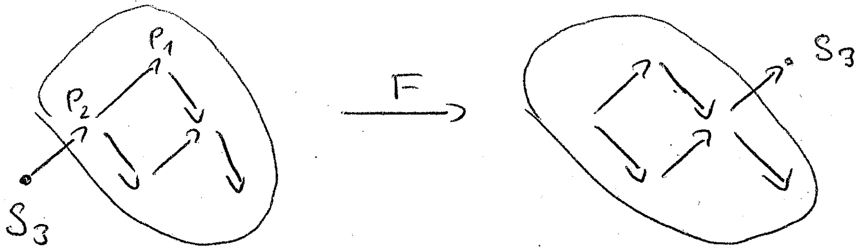
$\tau^6 \cong id$ $\qquad\qquad$ $\tau = id$

$\Omega^2 = id$

(4) Tilting Theory

(a) Module theoretic interpretation of BGP-reflection functors. *with Platzeck*

$Q: 1 \to 2 \to 3$ $\qquad\qquad$ $Q': 1 \to 2 \leftarrow 3$



$F = \mathrm{Hom}(T, -)$ where $T = P_1 \oplus P_2 \oplus \mathrm{TrD}(S_3)$

is "tilting module" (• pd $T \leq 1$

• $\mathrm{Ext}^1(T,T) = 0$

• $|T| = n$)

Idun Reiten
13.11.14

(b) <u>Relationship to contravariantly finite</u>

More general concept [Miyashita, Happel]

- $\operatorname{pd} T < \infty$
- $\operatorname{Ext}^i(T,T) = 0 \quad \forall i > 0$
- $\exists$ ex. sequence

$$0 \to \Lambda \to \underbrace{T_0 \to \cdots \to T_m}_{\in \operatorname{add} T} \to 0$$

<u>Theorem:</u> (gldim $< \infty$) There is a 1-1 correspondence

$$\text{basic cotilting modules} \longleftrightarrow \text{Contravariantly finite resolving subcategories}$$

$$T \longmapsto \{X \mid \operatorname{Ext}^i(X,T) = 0 \ \forall i > 0\}$$

$$\text{Ext-injectives in } \mathcal{X} \longleftarrow\!\shortmid \ \mathcal{X}$$

Development: Ringel's work on tilting modules for quasi-hereditary algebras.

(5) <u>General Nakayama Conjecture</u>

Λ f.d. algebra

$$0 \to \Lambda \to \underbrace{I_0 \to \cdots \to I_i \to \cdots}_{\text{injective resolution}}$$

I indec. injective, then $I \mid I_i$ some i

Equivalent:

$\Lambda \mid M$, $\operatorname{Ext}^i(M,M) = 0 \ \forall i > 0$, then M is projective

$\rightsquigarrow$ <u>Open</u>.