Tame Minimal Non-Polynomial Growth Simply Connected Algebras

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1 Introduction

By Drozd's Tame and Wild Theorem [8] the class of finite dimensional algebras (associative, with identity) over an algebraically closed field may be divided into two disjoint classes. One class consists of the tame algebras, for which the indecomposable modules occur in each dimension d in a finite number of discrete and a finite number of one-parameter families. The second class is formed by the wild algebras whose representation theory is as complicated as the study of finite dimensional vector spaces together with two non-commuting endomorphisms, for which the classification of the indecomposable finite dimensional modules is a well-known difficult problem. Hence we can hope to classify the modules only for tame algebras. Among the tame algebras we may distinguish the class of polynomial growth algebras A for which there exist integers m (depending on A) such that, in each dimension d, the indecomposable A-modules occur in a finite number of discrete and at most d^m one-parameter families.

Frequently, applying covering techniques, we may reduce the representation theory of a given tame (respectively, polynomial growth) algebra to that of the corresponding simply connected algebra. Recently, the class of polynomial growth simply connected algebras has been extensively investigated. In particular, a rather complete representation theory of polynomial growth strongly simply connected algebras has been established by the second author in [21]. One of the important open problems is to extend this theory to arbitrary simply connected algebras of polynomial growth. We are especially interested in criteria for a simply connected algebra to be of

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polynomial growth. This leads to the study of tame simply connected algebras which are minimally not of polynomial growth (they themselves are not of polynomial growth but every proper convex subcategory is).

The main aim of this article is to introduce and classify (by quivers and relations) a class of tame minimal non-polynomial growth simply connected algebras, which we call (generalized) pg-critical algebras. Moreover we describe basic properties of pg-critical algebras. In particular, we prove that the Euler (respectively Tits) quadratic form of any pg-critical algebra A is positive semi-definite of corank 2 and the eigenvalues of the Coxeter transformation of A are roots of unity. We also describe completely the structure of non-regular connected components of the Auslander-Reiten quivers of pg-critical algebras.

The paper is organized as follows. In section 2 we fix the notations and recall the definitions needed. In section 3 we introduce the polynomial growth critical algebras and classify them by quivers and relations. In particular, we prove that all such algebras are simply connected. In section 4 we determine the tilting classes of polynomial growth critical algebras and show that the Euler form of any such algebra is positive semi-definite with radical of rank 2. Section 5 is devoted to the spectral properties of polynomial growth critical algebras. We determine the Coxeter polynomial of any polynomial growth critical algebra and show that the spectral radius of its Coxeter matrix is 1. In the final section 6 we investigate the module category of polynomial growth critical algebras. Here, we describe the structure of all non-regular components of their Auslander Reiten quivers and discuss the behaviour of non-regular components in the category of indecomposable modules.

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2 Preliminaries

Throughout this article, K will denote a fixed algebraically closed field. By an algebra is meant an associative finite-dimensional K-algebra with an identity, which we shall assume to be basic and connected. An algebra A can be written as a bound quiver algebra $A \cong KQ/I$ where $Q = Q_A$ is the quiver of A and I is an admissible ideal in the path algebra KQ of Q. Equivalently, an algebra $A = KQ/I$ may be considered as a K-category whose object