Tame two-point algebras without loops

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Abstract

We determine the representation type of the algebras whose quiver has precisely two vertices and admits no loops by listing all maximal tame and minimal wild algebras of this form. It turns out that such an algebra A is tame if and only if $A/\mathrm{rad}^3 A$ is tame, and in this case A degenerates to a biserial algebra. Moreover, A is wild if and only if it is controlled wild.

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1 Introduction.

Let k be a fixed algebraically closed field. By an algebra we mean a finite-dimensional associative k-algebra with an identity, which we assume (without loss of generality) to be basic and connected. Let A be an algebra and Q be the quiver of A. Then we can write $A = kQ/I$ where I is an admissible ideal of path algebra kQ ([Ga1]).

The representation type of algebras is one of the main themes of representation theory of algebras. The representation type of local algebras, i.e. algebras whose quiver Q has just one vertex, has been completely determined ([BD,Br,D,Ge2,GP,HR,R1,R2]). Next, it is natural to determine the representation type of two-point algebras. The complete list of maximal representation-finite two-point algebras was given in [BG]. Concerning tame representation type of two-point algebras, there exist only partial answers so far: the tame triangular matrix algebras with two points were determined in [HM], and the tame two-point distributive algebras were classified in [HM], [Ge1] and [DG].

In the present paper, our aim is to determine the representation type of two-point algebras without loops, i.e. the algebras whose quiver $Q = Q(i, j)$

has two vertices 1 and 2, together with i arrows $\alpha_1, ..., \alpha_i$ from 1 to 2 and j arrows $\beta_1, ..., \beta_j$ from 2 to 1.

Denote by $k^n Q$ the ideal of kQ generated by the paths of length n. We say an algebra A is *of type* T_1 (resp. *of type* T_2) if A is, up to isomorphism and duality, of the form $kQ(2,1)/(\alpha_1\beta_1 + \sigma_1, \beta_1\alpha_1 + \sigma_2, \sigma_3, ..., \sigma_s, k^n Q(2,1))$ (resp. of the form $kQ(2,1)/(\alpha_1\beta_1 + \sigma_1, \beta_1\alpha_2 + \sigma_2, \sigma_3, ..., \sigma_s, k^n Q(2,1))$ with arbitrary elements $\sigma_p \in k^3 Q(2,1)$. Likewise, A is *of type* T_3 if it has (up to isomorphism and duality) the form $kQ(2,2)/(\alpha_1\beta_1 + \sigma_1, \alpha_2\beta_2 + \sigma_2, (x_1\beta_1 + x_2\beta_2)(y_1\alpha_1 + y_2\alpha_2) + \sigma_3, (x_3\beta_1 + x_4\beta_2)(y_3\alpha_1 + y_4\alpha_2) + \sigma_4, \sigma_5, ..., \sigma_s, k^n Q(2,2))$ with arbitrary elements $\sigma_p \in k^3 Q(2,2)$ and numbers $x_p, y_p \in k$ such that $x_1x_4 \neq x_2x_3$ and $y_1y_4 \neq y_2y_3$. The following theorems are the main results of this paper:

Theorem 1. *Let $A = kQ/I$ be a two-point algebra without loops. Then the following are equivalent:*
(1) A is tame;
(2) A is a factor of an algebra of type T_1, T_2 or T_3;
(3) A degenerates to a biserial algebra;
(4) $A/\mathrm{rad}^3 A$ is tame.

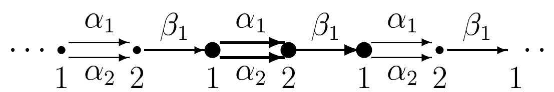
Theorem 2. *Let $A = kQ/I$ be a two-point algebra without loops. Then the following are equivalent:*
(1) A is wild;
(2) A has one of the algebras W_1, W_2, W_3 from the following list as a factor (up to isomorphism and duality);
(3) A is controlled wild.

	quiver Q	generators of the ideal I
W_1	$1 \bullet \overset{\alpha_1, \alpha_2, \alpha_3}{\Rrightarrow} \bullet 2$	
W_2	$1 \bullet \underset{\beta_1}{\overset{\alpha_1, \alpha_2}{\rightleftarrows}} \bullet 2$	$\beta_1\alpha_1, \beta_1\alpha_2.$
W_3	$1 \bullet \underset{\beta_1, \beta_2}{\overset{\alpha_1, \alpha_2}{\rightleftarrows}} \bullet 2$	$\beta_1\alpha_1, \beta_1\alpha_2, \beta_2\alpha_1, \beta_2\alpha_2, \alpha_1\beta_1, \alpha_1\beta_2 - \alpha_2\beta_1.$

2 Tameness and wildness

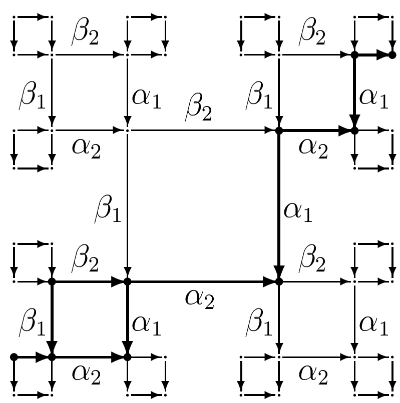
Lemma 1. *The algebras W_1, W_2 and W_3 are controlled wild.*

Proof. It is well-known that the algebra W_1 is wild hereditary. Using covering theory ([BG], [DS], [Ga2], [MP]), we will prove that the remaining two algebras are controlled wild. In case of W_2, we construct a Galois covering $\widetilde{W_2}$ with Galois group $\mathbb{Z}$ given by the quiver



and the relations $\alpha_1\beta_1 = 0 = \alpha_2\beta_1$. Then $\widetilde{W_2}$ has a wild hereditary algebra of type $\widetilde{\widetilde{\mathbb{A}_2}}$ as factor (indicated by thick lines in the figure above). By [H; Theorem (3.3)], the algebra W_2 is controlled wild with controlling index $c(W_2) \leq 1$.

For the algebra W_3, we construct the universal covering $\widetilde{W_3}$. Its Galois group is the free group (non-commutative) with two generators. The following figure shows only a finite part of the quiver of $\widetilde{W_3}$, and the relations are the same as for the algebra W_3.



Now $\widetilde{W_3}$ has a wild concealed algebra of type $\widetilde{\widetilde{\mathbb{E}_8}}$ as factor (indicated by thick lines in the figure above). By [H; Theorem (3.3)], the algebra W_3 is controlled wild with controlling index $c(W_3) \leq 17$. □

Lemma 2. *The algebras of type T_1, T_2 and T_3 degenerate to biserial algebras, and they are tame.*

Proof. Let A be of type T_1. Given an element $\delta \in k^3Q(2,1)$, we write $\delta = \sum_{m\geq 3}\delta_m$ where each δ_m is homogeneous of degree m (i.e., is a linear combination of paths of length m). Then for $\xi \in k$ we set $\delta(\xi) = \sum_{m\geq 3}\xi^{m-2}\delta_m$. Further, we define $\lambda(\xi) = \alpha_1\beta_1 + \sigma_1(\xi)$ and $\rho(\xi) = \beta_1\alpha_1 + \sigma_2(\xi)$. Finally, we denote by $A(\xi)$ the algebra $kQ(2,1)/J(\xi)$, where $J(\xi)$ is the ideal of $kQ(2,1)$ generated by $k^nQ(2,1)$ and $\lambda(\xi), \rho(\xi), \sigma_p(\xi), p \geq 3$. Clearly $A = A(1)$, and for $\xi \neq 0$ all the algebras $A(\xi)$ are isomorphic to A. Thus the algebra A degenerates to the algebra $A(0)$ which is a string algebra ([BR]). By Geiß's Theorem we have that A is tame. (Of course, we may also apply [CB; Theorem B]). Similarly, an algebra A of type T_2 also degenerates a string algebra, thus it is tame, and an algebra A of type T_3 degenerates to the algebra $A(0) = kQ(2,2)/(\alpha_1\beta_1, \alpha_2\beta_2, (x_1\beta_1+x_2\beta_2)(y_1\alpha_1+y_2\alpha_2), (x_3\beta_1+x_4\beta_2)(y_3\alpha_1+y_4\alpha_2), k^nQ(2,2))$ with $x_p, y_p \in k$, $x_1x_4 \neq x_2x_3$ and $y_1y_4 \neq y_2y_3$. By replacing $y_1\alpha_1 + y_2\alpha_2$ and $y_3\alpha_1 + y_4\alpha_2$ with α_1 and α_2, one sees that $A(0)$ is isomorphic to the algebra $T := kQ(2,2)/((z_1\alpha_1 + z_2\alpha_2)\beta_1, (z_3\alpha_1 + z_4\alpha_2)\beta_2, (x_1\beta_1 + x_2\beta_2)\alpha_1, (x_3\beta_1 + x_4\beta_2)\alpha_2, k^nQ(2,2))$. Choosing an appropriate bisection of the quiver $Q(2,2)$, the algebra T becomes a balanced homogeneous model biserial algebra, thus it is tame ([CB,VFCB]). $\square$

3. Classification

Recall that we are dealing with algebras with quiver $Q = Q(i,j)$. It is well-known that in the cases $(i,j) = (1,0)$, $(0,1)$ or $(1,1)$ the finite-dimensional algebras with underlying quiver $Q(i,j)$ are of finite representation type. Furthermore, for $(i,j) = (2,0)$ or $(0,2)$, the corresponding algebra $kQ(i,j)$ is just the Kronecker algebra, hence tame. If $i \geq 3$ or $j \geq 3$ then each algebra with quiver $Q(i,j)$ admits the wild algebra W_1 as a factor. Clearly, for all the cases above our theorems hold. Thus it remains to consider the cases $(i,j) = (2,1)$ and $(2,2)$ (The case $(i,j) = (1,2)$ is dual to the case $(i,j) = (2,1)$).

We say that the algebra $A' = kQ/I'$ is obtained from $A = kQ/I$ by *arrows replacement* if I' is obtained from I by substituting γ_1 and γ_2 with $x_1\gamma_1+x_2\gamma_2$ and $x_3\gamma_1 + x_4\gamma_2$ respectively, where γ_1 and γ_2 are two different arrows in Q with the same start point and end point, and x_p are elements in k satisfying $x_1x_4 - x_2x_3 \neq 0$. Moreover, we say that the algebra $A' = kQ/I'$ is obtained from $A = kQ/I$ by *relations replacement* if I' is obtained from I by replacing λ_1 and λ_2 with $x_1\lambda_1 + x_2\lambda_2$ and $x_3\lambda_1 + x_4\lambda_2$ respectively, where λ_1 and λ_2 are two generators of I, and x_p are elements in k satisfying $x_1x_4 - x_2x_3 \neq 0$. Clearly, in both cases the algebra A' is isomorphic to A.

First, we consider two-point algebras with quiver $Q(2,1)$.

Lemma 3. *Let $A = kQ/I$ be a two-point algebra with quiver $Q = Q(2,1)$. Then, up to isomorphism and duality, either A is a factor of an algebra of type T_1 or T_2, or A has the algebra W_2 as its factor.*

Proof. Denote by m (resp. n) the dimension of the k-vector space $(I/k^3Q) \cap ((kQ/k^3Q)(1,1))$ (resp. $(I/k^3Q) \cap ((kQ/k^3Q)(2,2))$). Note that $m = 2 - \dim_k \mathrm{rad}^2 P_A(1)/\mathrm{rad}^3 P_A(1)$, $n = 2 - \dim_k \mathrm{rad}^2 P_A(2)/\mathrm{rad}^3 P_A(2)$, and $0 \leq m, n \leq 2$, where $P_A(1)$ and $P_A(2)$ are the indecomposable projective A-modules corresponding to the vertices 1 and 2 respectively. Up to duality, we can assume that $m \leq n$. If $m = 0$ or $n = 0$ then A has the algebra W_2 or its dual as a factor.

Case $(m,n) = (1,1)$: Take $\lambda = x_1\alpha_1\beta_1 + x_2\alpha_2\beta_1 + \sigma_1$ to be an element in I with $x_1, x_2 \in k$ and $\sigma_1 \in k^3Q$ such that $\bar{\lambda} = x_1\bar{\alpha}_1\bar{\beta}_1 + x_2\bar{\alpha}_2\bar{\beta}_1$ is a k-basis of $(I/k^3Q) \cap ((kQ/k^3Q)(2,2))$. Through arrows replacement, namely replacing $x_1\alpha_1 + x_2\alpha_2$ with α_1, and replacing α_1 with α_2 in case $x_2 = 0$, we can choose the relation λ to be $\alpha_1\beta_1 + \sigma_1$. Take $\rho = y_1\beta_1\alpha_1 + y_2\beta_1\alpha_2 + \sigma_2$ to be an element in I with $y_1, y_2 \in k$ and $\sigma_2 \in k^3Q$ such that $\bar{\rho} = y_1\bar{\beta}_1\bar{\alpha}_1 + y_2\bar{\beta}_1\bar{\alpha}_2$ is a k-basis of $(I/k^3Q) \cap ((kQ/k^3Q)(1,1))$. If $y_2 \neq 0$, then replacing $x\alpha_1 + y\alpha_2$ with α_2 yields an algebra of type T_2. If $y_2 = 0$, then A is an algebra of type T_1.

Finally, for $n = 2$, the algebra A is a factor of a two-point algebra with quiver $Q(2,1)$ and $(m,n) = (1,1)$, therefore, up to isomorphism and duality, A is a factor of an algebra of type T_1 or T_2. $\square$

Next, we consider two-point algebras with quiver $Q(2,2)$

Lemma 4. *Let $A = kQ/I$ be a two-point algebra with quiver $Q = Q(2,2)$. Then, up to isomorphism and duality, either A is a factor of an algebra of type T_3, or A has the algebra W_2 or W_3 as a factor.*

Proof. Again denote by m (resp. n) the dimension of the k-vector space $(I/k^3Q) \cap ((kQ/k^3Q)(1,1))$ (resp. $(I/k^3Q) \cap ((kQ/k^3Q)(2,2))$). Up to duality, we can assume that $n \leq m$. Obviously $0 \leq m, n \leq 4$.

Case $(m,n) = (4,1)$: Since $m = 4$, the ideal I contains elements $\beta_1\alpha_1 + \sigma_1$, $\beta_1\alpha_2 + \sigma_2$, $\beta_2\alpha_1 + \sigma_3$ and $\beta_2\alpha_2 + \sigma_4$ for some $\sigma_p \in k^3Q$. Take $\rho = x_1\alpha_1\beta_1 + x_2\alpha_1\beta_2 + x_3\alpha_2\beta_1 + x_4\alpha_2\beta_2 + \sigma_5$ to be an element in I such that $\bar{\rho}$ is a k-basis of $(I/k^3Q) \cap ((kQ/k^3Q)(2,2))$. Through arrows replacement, it is easy to see that A has the algebra W_2 or W_3 as its factor. In fact, we distinguish the following two cases: If the vectors (x_1, x_2) and (x_3, x_4) are linearly dependent, then we replace the arrows β_1, β_2 such that ρ has the form $\rho = x_2'\alpha_1\beta_2 + x_4'\alpha_2\beta_2 + \sigma_5'$, and hence A has the algebra W_2 as factor. If (x_1, x_2) and (x_3, x_4) are linearly independent, then we apply an arrows replacement on β_1, β_2 such that ρ has the form $\rho = x_1'\alpha_1\beta_1 + x_2'\alpha_1\beta_2 + x_2'\alpha_2\beta_1 + \sigma_5'$. Then the algebra A has W_3 as factor.

Case $n \leq 1$: In this case, the algebra A has a two-point algebra with quiver $Q(2,2)$ and $(m,n) = (4,1)$ as factor, hence it admits one of W_2 or W_3 as its factor.

Case $(m,n) = (3,3)$: Let $\rho_1 = \beta_1\alpha_1 + x_1\beta_1\alpha_2 + \sigma_1$, $\rho_2 = \beta_2\alpha_2 + x_2\beta_1\alpha_2 + \sigma_2$ and $\rho_3 = \beta_2\alpha_1 + x_3\beta_1\alpha_2 + \sigma_3$ with $\sigma_p \in k^3Q$ be elements in I such that $\{\bar{\rho}_1, \bar{\rho}_2, \bar{\rho}_3\}$ is a k-basis of $(I/k^3Q) \cap ((kQ/k^3Q)(1,1))$. Through arrows replacement, we can choose $\rho_1 = \beta_1\alpha_1 + \sigma_1$ and $\rho_2 = \beta_2\alpha_2 + \sigma_2$. Further take elements λ_1, λ_2 and λ_3 in I such that $\{\bar{\lambda}_1, \bar{\lambda}_2, \bar{\lambda}_3\}$ is a k-basis of $(I/k^3Q) \cap ((kQ/k^3Q)(2,2))$. By relations replacement we obtain one of the following possibilities: Either $\lambda_1 = \alpha_1\beta_1 + y_1\alpha_2\beta_2 + \sigma_4$, $\lambda_2 = \alpha_1\beta_2 + y_2\alpha_2\beta_2 + \sigma_5$ and $\lambda_3 = \alpha_2\beta_1 + y_3\alpha_2\beta_2 + \sigma_6$, or $\lambda_1 = \alpha_1\beta_2 + y_1\alpha_1\beta_1 + \sigma_4$, $\lambda_2 = \alpha_2\beta_1 + y_2\alpha_1\beta_1 + \sigma_5$ and $\lambda_3 = \alpha_2\beta_2 + y_3\alpha_1\beta_1 + \sigma_6$, or $\lambda_1 = \alpha_1\beta_1 + y_1\alpha_2\beta_1 + \sigma_4$, $\lambda_2 = \alpha_1\beta_2 + y_2\alpha_2\beta_1 + \sigma_5$ and $\lambda_3 = \alpha_2\beta_2 + y_3\alpha_2\beta_1 + \sigma_6$, or $\lambda_1 = \alpha_1\beta_1 + y_1\alpha_1\beta_2 + \sigma_4$, $\lambda_2 = \alpha_2\beta_1 + y_2\alpha_1\beta_2 + \sigma_5$ and $\lambda_3 = \alpha_2\beta_2 + y_3\alpha_1\beta_2 + \sigma_6$. In any case, up to isomorphism and duality, A is a factor of an algebra of type T_3.

Case $n \geq 3$: In this case, the algebra A must be a factor of a two-point algebra with quiver $Q(2,2)$ and $(m,n) = (3,3)$, therefore, up to isomorphism and duality, A is a factor of an algebra of type T_3.

In the following, we always assume that $n = 2$, and we take elements λ_1, λ_2 in I such that $\{\bar{\lambda}_1, \bar{\lambda}_2\}$ is a k-basis of $(I/k^3Q) \cap ((kQ/k^3Q)(2,2))$. Up to isomorphism, we can choose the coefficient of $\alpha_1\beta_1$ in λ_1 to be 1. then we obtain the following possibilities: either $\lambda_1 = \alpha_1\beta_1 + x_1\alpha_2\beta_1 + x_2\alpha_2\beta_2 + \sigma_1$ and $\lambda_2 = \alpha_1\beta_2 + y_1\alpha_2\beta_1 + y_2\alpha_2\beta_2 + \sigma_2$, or $\lambda_1 = \alpha_1\beta_1 + x_1\alpha_1\beta_2 + x_2\alpha_2\beta_2 + \sigma_1$ and $\lambda_2 = \alpha_2\beta_1 + y_1\alpha_1\beta_2 + y_2\alpha_2\beta_2 + \sigma_2$, or $\lambda_1 = \alpha_1\beta_1 + x_1\alpha_1\beta_2 + x_2\alpha_2\beta_1 + \sigma_1$ and $\lambda_2 = \alpha_2\beta_2 + y_1\alpha_1\beta_2 + y_2\alpha_2\beta_1 + \sigma_2$. Through arrows replacement and relations replacement, we can show that either A has the algebra W_2 or W_3 as a factor, or that the relations have the form $\lambda_1 = \alpha_1\beta_1 + \sigma_1$ and $\lambda_2 = \alpha_2\beta_2 + \sigma_2$.

We suppose from now on that $\lambda_1 = \alpha_1\beta_1 + \sigma_1$ and $\lambda_2 = \alpha_2\beta_2 + \sigma_2$. If $m \geq 3$ then it is easy to see that A must be a factor of an algebra of type T_3. Thus we only need consider the case $m = 2$. Take elements ρ_1 and ρ_2 in I such that $\{\bar{\rho}_1, \bar{\rho}_2\}$ is a k-basis of $(I/k^3Q) \cap ((kQ/k^3Q)(1,1))$. The following cases have to be considered: $\rho_1 = \beta_1\alpha_1 + x_1\beta_2\alpha_1 + x_2\beta_2\alpha_2 + \sigma_3$ and $\rho_2 = \beta_1\alpha_2 + y_1\beta_2\alpha_1 + y_2\beta_2\alpha_2 + \sigma_4$, or $\rho_1 = \beta_1\alpha_1 + x_1\beta_1\alpha_2 + x_2\beta_2\alpha_2 + \sigma_3$ and $\rho_2 = \beta_2\alpha_1 + y_1\beta_1\alpha_2 + y_2\beta_2\alpha_2 + \sigma_4$, or $\rho_1 = \beta_1\alpha_1 + x_1\beta_1\alpha_2 + x_2\beta_2\alpha_1 + \sigma_3$ and $\rho_2 = \beta_2\alpha_2 + y_1\beta_1\alpha_2 + y_2\beta_2\alpha_1 + \sigma_4$. As above, by arrows replacement and relations replacement one can show that either A has the algebra W_2 or W_3 as a factor, or A is an algebra of type T_3. □

Example. We consider the algebra $A = kQ(2,2)/(\alpha_1\beta_1 - \alpha_2\beta_2, \alpha_2\beta_1 - \alpha_1\beta_2, \beta_1\alpha_1, \beta_2\alpha_2, k^nQ(2,2))$. Denote by ρ and λ the relations $\alpha_1\beta_1 - \alpha_2\beta_2$

and $\alpha_2\beta_1 - \alpha_1\beta_2$ respectively. Replacing $\rho + \lambda$ by ρ, we can choose $\rho = (\alpha_1 + \alpha_2)(\beta_1 - \beta_2)$. In case char $k = 2$, we rewrite λ as $\lambda = \alpha_2(\beta_1 - \beta_2) - (\alpha_1 + \alpha_2)\beta_2$ and, replacing $\alpha_1 + \alpha_2$ and $\beta_1 - \beta_2$ with α_1 and β_1 respectively, one sees that the algebra A has a factor which is isomorphic to the dual of W_3, hence A is wild. In case char$k \neq 2$, however, replacing $2\lambda - \rho$ with λ, we have $\lambda = (-\alpha_1 + \alpha_2)(\beta_1 + \beta_2)$, i.e. the algebra A is of typ.e T_3, thus it is tame. This example also shows that if an algebra with quiver $Q(2,2)$ is not of the form as the algebras W_p and T_p then its representation type depends on the characteristic of the field k.

Proof of Theorem 1. By Lemma 1, Lemma 2, Lemma 3 and Lemma 4, we have (1) $\Leftrightarrow$ (2) $\Leftrightarrow$ (3). (1) $\Rightarrow$ (4) is trivial. Note that all W_1, W_2 and W_3 are radical cube zero. Therefore if A is wild then $A/\mathrm{rad}^3 A$ is wild, i.e. (4) $\Rightarrow$ (1). $\square$

Proof of Theorem 2. By Lemma 1, Lemma 2, Lemma 3 and Lemma 4, we have (1) $\Rightarrow$ (2) $\Rightarrow$ (3). By [H; Proposition (2.2)], (3) $\Rightarrow$ (1). $\square$

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