Minimal representations of conformal groups and generalized Laguerre functions

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We give a unified construction for the minimal representation of the conformal group G associated to a simple real (non necessarily euclidean) Jordan algebra V. This representation is realized on the L^2-space of the minimal orbit $\mathcal{O}$ of the structure group L of V. We construct its corresponding $(\mathfrak{g}, K)$-module and show that it can be integrated to a unitary irreducible representation of G on $L^2(\mathcal{O})$.

This minimal representation is thought to correspond to the minimal coadjoint orbit. In particular, we obtain a unified approach to the L^2-models of the two most prominent minimal representations, namely the Shale–Segal–Weil representation of the metaplectic group $\mathrm{Mp}(n, \mathbb{R})$ and the minimal representation of $\mathrm{O}(p, q)$ which was recently studied by T. Kobayashi, G. Mano and B. Ørsted.

Finally we find an explicit expression in terms of Bessel functions for the functions in every K-type which are invariant under a maximal compact subgroup M of the structure group L. Various properties of these special functions such as a fourth order differential equation, recurrence relations and integral formulas connect to the representation theory involved. In the case where V is a euclidean Jordan algebra (i.e. G/K is hermitian symmetric of tube type), these functions are generalized Laguerre functions on the Jordan algebra V as studied by J. Faraut and A. Koranyi.