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# $G$-prime ideals in $\mathfrak{L}^1$-convolution algebras of nilpotent Lie groups

Let $N$ be a simply connected nilpotent Lie group with Lie algebra $\mathfrak{n}$. The integrable functions on $N$ form an involutive Banach algebra $\mathfrak{L}^1(N)$ under convolution, whose primitive (= maximal) spectrum Priv $\mathfrak{L}^1(N)$ can be identified with the orbit space $N \setminus \mathfrak{n}^*$, where $\mathfrak{n}^*$ denotes the real linear dual of $\mathfrak{n}$. Each closed subset $A$ of Priv $\mathfrak{L}^1(N)$ gives rise to closed ideal $k(A)$ in $\mathfrak{L}^1(N)$,$k(A) = \bigcap_{I \in A} I$. Any group $G$ acting by automorphisms on $N$ also operates on $\mathfrak{L}^1(N)$ and on Priv $\mathfrak{L}^1(N)$. A closed two-sided $G$-invariant ideal $P$ in $\mathfrak{L}^1(N)$ is called $G$-prime, if for all $G$-invariant two-sided ideals $I$ and $J$ in $\mathfrak{L}^1(N)$ the inclusion $I * J \subset P$ implies that $I$ or $J$ is contained in $P$.

If $G$ happens to be a semidirect product of a compact Lie group and another connected nilpotent Lie group (acting unipotently on $\mathfrak{n}$) then the $G$-prime ideals are what they should be, namely of the form $k(A)$ for $G$-orbits $A$ in Priv $\mathfrak{L}^1(N)$.

The proof is based on a proposition for transformation groups and on "synthesis properties". Synthesis is more familiar in the context of commutative harmonic analysis, especially on Euclidean spaces. Here we need some non-commutative variant.