Let G be a simple Lie group and $\mathfrak{g}$ its Lie algebra. A finite order (let us say m) automorphism θ of G defines a $\mathbb{Z}/m\mathbb{Z}$ grading: $\mathfrak{g} = \bigoplus_{i=0}^{m} \mathfrak{g}_i$. If G_0 is the identity component of G^θ, then it acts on $\mathfrak{g}_1$ and this action is called a θ-representation. We are interested in some representations of stabilisers $G_{0,v}$ for $v \in \mathfrak{g}_1$, or more precisely, in a certain equality for related index.

Let Q be an algebraic group with the Lie algebra $\mathfrak{q}$ and V a finite-dimensional Q-module. Recall that the index of V, denoted $\mathrm{ind}\,(\mathfrak{q}, V)$, is the minimal codimension of the Q-orbits in the dual space V^*. There is a general inequality, due to Vinberg, which states that $\mathrm{ind}\,(\mathfrak{q}, V^*) \leqslant \mathrm{ind}\,(\mathfrak{q}_v, (V/\mathfrak{q}{\cdot}v)^*)$ for $v \in V$. (Note that here $V/\mathfrak{q}{\cdot}v$ is a $\mathfrak{q}_v$-module.) A pair (Q, V) is said to have GIB if Vinberg's inequality turns into an equality for all $v \in V$. We will present classification of inner automorphisms of $\mathfrak{gl}_n$ and all finite order autmorphisms of the exceptional Lie algebras such that $(G_0, \mathfrak{g}_1)$ has GIB and $\mathfrak{g}_1$ contains a non-zero semisimple element.

This is a joint project with Willem de Graaf.