Lie Groups and Algebraic Groups

24–25 July 2014

Faculty of Mathematics
Bielefeld University
Lecture Room: V3-201

This workshop is part of the DFG-funded CRC 701
Spectral Structures and Topological Methods in Mathematics
at Bielefeld University

Organisers: Herbert Abels and Ernest Vinberg

http://www.math.uni-bielefeld.de/sfb701/2014_LieGroups

Schedule

Thursday, July 24th, 2014
Lecture Room: **V3-201**

10:00 – 10:50	**Dmitri Panyushev** (Moscow) Orbits of a Borel subgroup in abelian ideals and the Pyasetskii correspondence
11:00 – 11:50	**Ghislain Fourier** (Glasgow) PBW filtrations, posets and symmetric functions
11:50 – 12:30	**Coffee Break**
12:30 – 13:20	**Christian Lange** (Cologne) Classification and characterization of pseudoreflection groups
13:20 – 15:30	**Lunch Break**
15:30 – 16:20	**Peter Heinzner** (Bochum) Geometry of actions of real reductive groups on Kählerian manifolds
16:30 – 17:20	**Hannah Bergner** (Bochum) Conjugacy classes of n-tuples in semi-simple Jordan algebras
17:20 – 18:00	**Coffee Break**
18:00 – 18:50	**Bjorn Villa** (Bochum) On the geometry of the convex hull of noncompact hermitian coadjoint orbits

Schedule

Friday, July 25th, 2014
Lecture Room: **V3-201**

09:30 – 10:20	**Leonid Potyagailo** (Lille) Similar relatively hyperbolic actions of a group
10:30 – 11:20	**Werner Hoffmann** (Bielefeld) The trace formula and prehomogeneous vector spaces
11:20 – 12:00	**Coffee Break**
12:00 – 12:50	**Konrad Schöbel** (Jena) Separation coordinates and moduli spaces of stable curves (joint work with Alexander P. Veselov)
12:50 – 15:00	**Lunch Break**
15:00 – 15:50	**Detlev Poguntke** (Bielefeld) Kirillov theory without polarizations
16:00 – 16:50	**Karl Hofmann** (Darmstadt) On the Chabauty space of locally compact groups
16:50 – 17:30	**Coffee Break**
17:30 – 18:20	**Willem de Graaf** (Trento) Regular subalgebras and nilpotent orbits in real graded Lie algebras

Abstracts

Dmitri Panyushev (Moscow)

Orbits of a Borel subgroup in abelian ideals and the Pyasetskii correspondence

Let B be a Borel subgroup of a semisimple algebraic group G, and let $\mathfrak{a}$ be an abelian ideal of $\mathfrak{b}=Lie(B)$. The ideal $\mathfrak{a}$ is determined by a certain subset $\Delta_{\mathfrak{a}}$ of positive roots, and using $\Delta_{\mathfrak{a}}$ we give an explicit classification of the B-orbits in $\mathfrak{a}$ and $\mathfrak{a}^*$. Our description visibly demonstrates that there are finitely many B-orbits in both cases. Then we describe the Pyasetskii correspondence between the B-orbits in $\mathfrak{a}$ and $\mathfrak{a}^*$ and the invariant algebras $k[\mathfrak{a}]^U$ and $k[\mathfrak{a}^*]^U$, where $U=(B,B)$. As an application, the number of B-orbits in the abelian nilradicals is computed. We also discuss related results of A. Melnikov and others for classical groups and state a general conjecture on the closure and dimension of the B-orbits in the abelian nilradicals, which exploits a relationship between between B-orbits and involutions in the Weyl group.

Ghislain Fourier (Glasgow)

PBW filtrations, posets and symmetric functions

I will recall the PBW filtration on cyclic modules for a complex Lie algebra and the results known for irreducible modules for simple complex Lie algebras. Then I will generalize the construction of the graded modules and apply it to cyclic modules for the truncated current algebra. This will link conjectures on fusion products to conjectures on posets of symmetric functions.

Christian Lange (Cologne)

Classification and characterization of pseudoreflection groups

A pseudoreflection group is a finite linear group over the real numbers generated by transformations with codimension two fixed point subspace. Such groups naturally arise in the theory of orbifolds and are closely related to reflection groups. We explain their classification and characterize them in terms of quotient spaces.

Peter Heinzner (Bochum)

Geometry of actions of real reductive groups on Kählerian manifolds

t.b.a.

Hannah Bergner (Bochum)

Conjugacy classes of n-tuples in semi-simple Jordan algebras

Let J be a (complex) semi-simple Jordan algebra, and consider the action of the automorphism group on the n-fold product of J via the diagonal action. In the talk, geometric properties of this action are studied. In particular, a characterization of the closed orbits is given. In the case of a complex reductive linear algebraic group and the adjoint action on its Lie algebra, the closed orbits are precisely the orbits through semi-simple elements. More generally, a result of R.W. Richardson characterizes the closed orbits of the diagonal action on the n-fold product of the Lie algebra. A similar condition can be found in the case of Jordan algebras. It turns out that the orbit through an n-tuple $x=(x_1,...,x_n)$ is closed if and only if the Jordan subalgebra generated by $x_1,...,x_n$ is semi-simple.

Bjorn Villa (Bochum)

On the geometry of the convex hull of noncompact hermitian coadjoint orbits

We will study Kahlerian coadjoint orbits of Hermitian Lie groups (e.g. the real symplectic group $Sp(n,R)$, $SU(p,q)$ and others) and their convex hulls. It is known that the momentum images of these orbits in a compact Cartan subalgebra are convex polyhedra. Furthermore we will see that the faces of the convex hull of an orbit are exposed and are given by the face structure of the momentum image.

Leonid Potyagailo (Lille)

Similar relatively hyperbolic actions of a group

This is a joint work with Victor Gerasimov (University of Belo Horisonte, Brasil). Let a discrete group G possess two convergence actions by homeomorphisms on compacta X and Y. Consider the following question: does there exist a convergence action of G on a compactum Z and continuous equivariant maps $X \leftarrow Z \rightarrow Y$? We call the space Z (and action of G on it) pullback space (action). In such general setting a negative answer follows from a recent result of O. Baker and T. Riley. Suppose, in addition, that the initial actions are relatively hyperbolic that is they are non-parabolic and the induced action on the space of distinct pairs of points is cocompact. In the case when G is finitely generated the universal pullback space exists by a theorem of V. Gerasimov. We show that the situation drastically changes already in the case of countable non-finitely generated groups. We provide an example of two relatively hyperbolic actions of the free group G of countable rank for which the pullback action does not exist. Our main result is that the pullback space exists for two relatively hyperbolic actions of any group G if and only if the maximal parabolic subgroups of one of the actions are dynamically quasiconvex for the other one. We study an analog of the geodesic flow for a large subclass of convergence groups including the relatively hyperbolic ones. The obtained results imply the main result and seem to have an independent interest.

Werner Hoffmann (Bielefeld)

The trace formula and prehomogeneous vector spaces

The geometric side of the Arthur-Selberg trace formula expresses a certain distribution $J(f)$ on an adelic reductive group as a sum of integrals of the test function f over conjugacy classes with respect to certain non-invariant measures. Those measures are known only in special cases. I will present an approach to express them in terms of prehomogeneous zeta integrals. This has been realised for groups of rank up to 2. The problem is that $J(f)$ is defined as a sum indexed by cosets in parabolic subgroups with respect to their unipotent radicals, which is incompatible with the decomposition into conjugacy classes. The rearrangement uses induction of conjugacy classes, Siegel's mean value formula and canonical parabolics.

Konrad Schöbel (Jena)

Separation coordinates and moduli spaces of stable curves (joint work with Alexander P. Veselov)

We establish a surprising link between two a priori completely unrelated objects: The space of isometry classes of separation coordinates for the Hamilton-Jacobi equation on an n-dimensional sphere one one hand and the Deligne-Mumford moduli space $M_{0,n+2}$ of stable algebraic curves of genus zero with $n{+}2$ marked points on the other hand. This relation is proved by realising separation coordinates as maximal abelian subalgebras in a representation of the Kohno-Drinfeld Lie algebra. We use the rich combinatorial structure of $M_{0,n+2}$ and the closely related Stasheff polytopes in order to classify the different canonical forms of separation coordinates. Moreover, we infer an explicit construction for separation coordinates and the corresponding quadratic integrals from the mosaic operad on $M_{0,n+2}$.

Detlev Poguntke (Bielefeld)

Kirillov theory without polarizations

In his seminal doctoral dissertation Kirillov constructed a bijection between the unitary dual $\hat{N}$ of a simply connected nilpotent Lie group N and the orbit space $\mathfrak{n}^*/N$ (orbit method). Later, Bernat extended this result to exponential groups G.

Traditionally one constructs a map $\mathfrak{g}^*/G \to \hat{G}$ using (Pukanszky) polarizations. It requires some work to see the independence of the chosen polarizations.

Here we construct a family of bijections $\kappa_G : \hat{G} \to \mathfrak{g}^*/G$, G an exponential Lie group, in the opposite direction. This construction depends on the choice of an abelian ideal, which in a way is a milder arbitrariness. But still the independence has to be established.

Furthermore, the family (κ_G) is canonical in the sense that it can be (uniquely) characterized in terms of a short list of plausible properties. If one restricts to nilpotent groups one has, with some extra work, an even nicer characterization of this family (κ_N).

Karl Hofmann (Darmstadt)

On the Chabauty space of locally compact groups

The set $\mathcal{SUB}(G)$ of all closed subgroups of any locally compact group G carries a canonical compact Hausdorff topology (nowadays called *Chabauty topology*). In order to sample recent interest in this functorial parameter, Let $\mu_G\colon G \to \mathcal{SUB}(G)$ denote the function which attaches to an element g of G the closed subgroup $\%\langle g\rangle$ generated by it. It is shown that G is totally disconnected if and only if μ_G is continuous. Other functions $G \to \mathcal{SUB}(G)$ which associate with an element of G in a natural way a closed subgroup of G are

$$\begin{aligned} \mathrm{lev}_G(g) &= \big\{x \in G \,|\, \{g^k x g^{-k}\}_{k\in\mathbf{Z}} \text{ is precompact}\big\}, \text{ the } \textit{Levi} \text{ subgroup of } g\text{; and} \\ \mathrm{par}_G(g) &= \big\{x \in G \,|\, \{g^k x g^{-k}\}_{k\in\mathbf{N}} \text{ is precompact}\big\}, \text{ the } \textit{parabolic} \text{ subgroup of } g. \end{aligned}$$

They are shown to be continuous for totally disconnected G. Other functions $G \to \mathcal{SUB}(G)$ which are natural fail to be continuous even if G is totally disconnected. (Joint work with George W. Willis)

Willem de Graaf (Trento)

Regular subalgebras and nilpotent orbits in real graded Lie algebras

Dynkin has given an algorithm to classify the regular semisimple subalgebras of a complex semisimple Lie algebra, up to conjugacy by the inner automorphism group. Here we show how this can be extended to semisimple Lie algebras defined over the real numbers. Vinberg has shown that a classification of a certain type of regular subalgebras (called carrier algebras) in a graded semisimple Lie algebra, yields a classification of the nilpotent orbits in a homogeneous component of the graded Lie algebra. Our methods can also be used to classify the carrier algebras in a real graded semisimple Lie algebra. At the end we will discuss what needs to be done to obtain a classification of the nilpotent orbits from that. Such classifications of nilpotent orbits have applications in differential geometry and in theoretical physics.

Participant List

H. Abels	(Bielefeld)
H. Bergner	(Bochum)
A. Elashvili	(Tbilisi)
G. Fourier	(Glasgow)
W. d. Graaf	(Dipartimento d. Matematica, Trento)
P. Heinzner	(Bochum)
W. Hoffmann	(Bielefeld)
K. H. Hofmann	(Darmstadt)
Ch. Lange	(Cologne)
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D. Poguntke	(Bielefeld)
L. Potyagailo	(Lille)
K. Schöbel	(Jena)
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W. Tsanov	(Bochum)
B. Villa	(Bochum)
E. Vinberg	(Mosco. State University)
O. Yakimova	(Jena)