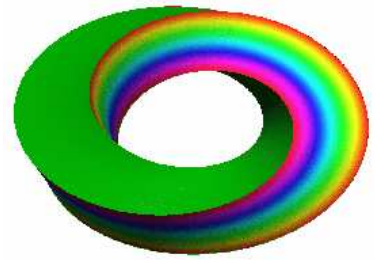**Universität Bielefeld**

Fakultät für Mathematik

Workshop on Free Probability and Random Combinatorial Structures

7 − 9 December 2009

Department of Mathematics

University of Bielefeld

V3-201 (Common Room)

This workshop is part of the conference program of the DFG-funded CRC 701

Spectral Structures and Topological Methods in Mathematics

at the University of Bielefeld

Organizer: Friedrich Götze

http://www.math.uni-bielefeld.de/sfb701/workshops/2009_FreeProbability/index.html

Programme

Monday, 7 December 2009

9:00 – 10:00   **Marek Bozejko** (University of Wroclaw)
Positive definite functions on permutation groups, Generalized Brownian motions with applications to free probability

10:00 – 10:25   *Coffee break*

10:25 – 11:25   **Mireille Capitaine** (Université de Toulouse)
Free probability and extreme eigenvalues of a large deformed Wigner matrix

11:30 – 12:30   **Nizar Demni** (École Nationale d'ingenieurs de Bizerte)
Ultraspherical polynomials and generalized Cauchy-Stieltjes transforms

12:30 – 14:00   *Lunch break*

14:00 – 15:00   **Franz Lehner** (Technische Universität Graz)
The Normal Law, Free Probability and a Hopf Algebra of Rooted Binary Trees

15:05 – 16:05   **Uwe Franz** (Université de Franche-Comté)
On monotone convolutions and monotone infinite divisibility

16:05 – 16:30   *Coffee break*

16:30 – 17:30   **Gennadii Chistyakov** (Universität Bielefeld)
Limit theorems for sums of free random variables

Tuesday, 8 December 2009

9:00 – 10:00   **Roland Speicher** (Queen's University)
Quantum Symmetries in Free Probability

10:00 – 10:25   *Coffee break*

10:25 – 11:25   **Mikael de la Salle** (École normale supérieure)
Strong Haagerup inequalities with operator coefficients

11:30 – 12:30   **Valentin Féray** (Université Bordeaux 1)
Large random Young diagrams and representation theory

12:30 – 14:00   *Lunch break*

14:00 – 15:00   **Piotr Śniady** (University of Wroclaw)
Characters of symmetric groups and free cumulants

15:05 – 16:05   **Jonathan Novak** (Queen's University)
Jucys-Murphy elements and unitary matrix integrals

16:05 – 16:30   *Coffee break*

16:30 – 17:30   **Holger Kösters** (Universität Bielefeld)
Characteristic Polynomials of Sample Covariance Matrices

19:00 –   Joint dinner at Bültmannshof
Bültmannshof, Kurt-Schumacher-Straße 17a, 33615 Bielefeld

(Please note: For the dinner prior registration is required.)

Wednesday, 9 December 2009

9:00 – 10:00   **Hans Maassen**   (Radboud University, Nijmegen)
Asymptotic behavior of quantum systems under repeated or continuous observation

10:00 – 10:25   *Coffee break*

10:25 – 11:25   **Ildar Ibragimov** (Academy of Sciences, St. Petersburg)
Dmitry Zaporozhets (Steklov Institute, St. Petersburg)
On roots of random polynomials

11:30 – 12:30   **Alexander Tikhomirov** (Komi Research Centre of RAS, Syktyvkar)
On the distribution of singular values and eigenvalues of products and powers of large non-hermitian matrices

12:30 – 14:00   *Lunch break*

14:00 – 15:00   **Mindaugas Bloznelis**   (Vilnius University)
Random intersection graphs with asymptotically independent edges: introduction, phase transition, small world

15:05 – 16:05   **Vidmantas Bentkus** (Institute of Math. and Informatics, Vilnius)
Inequalities for martingales and supermartingales with unbounded differences

16:05 – 16:30   *Coffee break*

16:30 – 17:30   **Andrei Zaitsev** (Steklov Institute, St. Petersburg)
Estimates of the rate of convergence in the Central Limit Theorem

17:30 –   *Closing of the workshop and coffee*

Abstracts

Vidmantas Bentkus (Institute of Math. and Informatics, Vilnius)

Inequalities for martingales and supermartingales with unbounded differences

Mindaugas Bloznelis (Vilnius University)

Random intersection graphs with asymptotically independent edges: introduction, phase transition, small world

I start with a simple introduction to the random intersection graph model. Then I will describe the phase transition (emergence of a giant connected component) in terms of survival probabilities of the related branching process. If time permits I shall show $O(\ln \ln n)$ upper bound in probability for the typical distance between vertices in the random intersection graph with power law degree distributions (without finite second moment).

Marek Bozejko (University of Wroclaw)

Positive definite functions on permutation groups, Generalized Brownian motions with applications to free probability

We introduce some classes of positive definite functions on permutation groups $S(n)$ related to the number of cycles , the number of connected components (i.e.the number of different Coxeter generators) and also others connected with Thoma characters on $S(\infty)$. Then we construct new models of Generalized Brownian motions extending the results of M. Guta, H. Maassen, R. Speicher and myself. Applications to construction of new infinitely divisible laws in the free additive convolutions will be also presented-among others Normal law $N(0, 1)$ and others.
Connections with the noncommutative Khinchine inequality will be also done.

Mireille Capitaine (Université de Toulouse)

Free probability and extreme eigenvalues of a large deformed Wigner matrix

We will show how free probability theory sheds light on the transition phase phenomenon which occurs at the convergence level of the extreme eigenvalues of a large deformed Wigner matrix.

Gennadii Chistyakov (Universität Bielefeld)

Limit theorems for sums of free random variables

Nizar Demni (École Nationale d'ingenieurs de Bizerte)

Ultraspherical polynomials and generalized Cauchy-Stieltjes transforms

We characterize, up to a conjecture, ultraspherical type generating functions for orthogonal polynomials. As a by product we obtain a new approach to the free Meixner family using differential equations. Then, we derive relations between generalized Cauchy Stieltjes transforms and powers of ordinary ones. A particular case gives a probability distribution interpolating the arcsine and the Wigner distributions. The even moments of this symmetric distribution are expressed as hypergeometric functions and generalize Catalan and Shifted Catalan numbers. Some numerical values will be provided and some similarities with generalized Gamma convolutions will be shown.

Valentin Féray (Université Bordeaux 1)

Large random Young diagrams and representation theory

The asymptotic behaviour of large Young diagrams under Plancherel measure have been studied since the 70's, when a limit law has been found, showing an unexpected relation with large random matrices. Recent developments in different directions allow to give a simple proof and generalize this result: first, the theory of "polynomial functions on the set of Young diagrams" by S. Kerov and G. Olshanski and second, the links between representation theory of symmetric groups and free probability (via the combinatorics of free cumulants) found by P. Biane. We will explain all these ideas and see how to adapt them to look at large, but not balanced Young diagrams : this gives new results on the asymptotics of q-Plancherel measure (a natural deformation of Plancherel measure introduced by E. Stahov).

Uwe Franz (Université de Franche-Comté)

On monotone convolutions and monotone infinite divisibility

Monotone independence and the associated monotone convolution for probability measures on the real line were introduced by Muraki. In this talk I will give a survey of the basic theory of monotone convolutions and the characterization of monotonically infinitely divisible probability measures. I will also discuss their connection to free convolutions and to Loewner chains.

Ildar Ibragimov (Academy of Sciences, St. Petersburg)
Dmitry Zaporozhets (Steklov Institute, St. Petersburg)

On roots of random polynomials

Let $\xi_0, \xi_1, \ldots, \xi_n, \ldots$ be a sequence of random variables. We assume that these variables are independent, identically distributed, and nondegenerate.
Consider a random polynomial of one variable

$$G_n(t) = \xi_0 + \xi_1 t + \ldots + \xi_{n-1} t^{n-1} + \xi_n t^n.$$

We consider two natural questions: how many roots of G_n are real in average and what is the asymptotic distribution of complex roots of G_n?
Especial attention is given to the recent result concerning the case when there are no conditions on the distribution of ξ_i.

Holger Kösters (Universität Bielefeld)

Characteristic Polynomials of Sample Covariance Matrices

In joint work with F. Götze we recently showed that the correlation coefficient of the characteristic polynomial of a general Wigner matrix is asymptotically given by the sine kernel. In this talk I will discuss some similar results for sample covariance matrices.

Franz Lehner (Technische Universität Graz)

The Normal Law, Free Probability and a Hopf Algebra of Rooted Binary Trees

We have shown that the normal distribution is infinitely divisible with respect to Voiculescu's free convolution. The proof is analytic, but the problem has interesting combinatorial ramifications which are not well understood yet and will be surveyed in this talk. This is joint work with S. Belinschi, M. Bozejko and Roland Speicher.

Hans Maassen (Radboud University, Nijmegen)

Asymptotic behavior of quantum systems under repeated or continuous observation

When a quantum system is being observed repeatedly or continuously, the expected values of its observables, conditioned on the observations made up to time t, determine a state. When t runs from 0 to infinity, a stochastic process is formed with values in the system's state space. The paths of this process are know as "quantum trajectories". We study their asymptotic behavior with a view on ergodicity and tendency toward pure states. Ergodicity holds under reasonable conditions, but

in a surprising way. Even when perfect observations are performed, purification is not certain since the system may get caught in a 'dark subspace', yielding no further information.

Jonathan Novak (Queen's University)

Jucys-Murphy elements and unitary matrix integrals

We show that many important properties of unitary matrix integrals, such as large N expansions, character expansions, and in some cases even explicit formulas, can be obtained from properties of Jucys-Murphy elements. The class of integrals to which our results apply are the correlation functions of elements of Haar-distributed random unitary matrices. In the course of our study we obtain various new results related to the conjugacy class expansion of symmetric functions of Jucys-Murphy elements, a topic of interest in algebraic combinatorics. This is joint work with Sho Matsumoto from Nagoya University.

Mikael de la Salle (École normale supérieure)

Strong Haagerup inequalities with operator coefficients

Haagerup's inequality is an estimate for the operator norm of d-homogeneous polynomials in the free group factors, and has had numerous generalizations. In this talk we present a Haagerup inequality with operator coefficients in Kemp and Speicher's holomorphic setting. Namely we shall establish an inequality for holomorphic d-homogeneous polynomials $\sum_{i=(i_1,\dots,i_d)} a_i \otimes \lambda(g_{i_1} \dots g_{i_d})$ with operator-valued coefficients $a_{i_1,\dots,i_d}$, in terms of the $d+1$ non-crossing labellings $M_k = (a_{(i_1,\dots,i_k),(i_{k+1},\dots,i_n)})$ as a $\mathbf{N}^k \times \mathbf{N}^{d-k}$ matrix of operators. Such inequalities are valid for the operator norm as well as the L^p norm (p an even integer), and in the more general setting of R-diagonal operators.

Piotr Śniady (University of Wroclaw)

Characters of symmetric groups and free cumulants

I am interested in representation theory of the symmetric groups S_n in the limit when n tends to infinity. It was observed by Kerov and Biane that the asymptotics of (normalized) irreducible characters corresponding to Young diagrams with a big number of boxes is given by free cumulants. Kerov observed that free cumulants can be used not only to give asymptotic, but also exact formulas for characters, thanks to Kerov character polynomials. Kerov polynomials seem to have a difficult structure, which at present is understood only partially. In my talk I will present recent developments in combinatorial understanding of Kerov polynomials and some open problems.

Roland Speicher (Queen's University)

Quantum Symmetries in Free Probability

In recent years it has become increasingly apparent that quantum symmetries (i.e., the invariance under the action of some quantum group) play an important role in free probability. The starting point of this was my joint work with Claus Koestler on a free de Finetti Theorem, where we showed that invariance under the action of quantum permutations characterizes freeness with amalgamation, for an infinite sequence of random variables. More general results in this direction for a recently introduced class of quantum groups (called "easy") were obtained in joint work with Teo Banica and Stephen Curran. I will survey some of these developments.

Alexander Tikhomirov (Komi Research Centre of RAS, Syktyvkar)

On the distribution of singular values and eigenvalues of products and powers of large non-hermitian matrices

Andrei Zaitsev (Steklov Institute, St. Petersburg)

Estimates of the rate of convergence in the Central Limit Theorem

Registered participants

Vidmantas Bentkus   (Institute of Math. and Informatics, Vilnius)
Mindaugas Bloznelis   (Vilnius University)
Marek Bozejko   (University of Wroclaw)
Jan Bücken   (Universität Bielefeld)
Mireille Capitaine   (Université de Toulouse)
Gennadii Chistyakov   (Universität Bielefeld)
Nizar Demni   (École Nationale d'ingenieurs de Bizerte)
Guido Elsner   (Universität Bielefeld)
Valentin Féray   (Université Bordeaux 1)
Uwe Franz   (Université de Franche-Comté)
Barbara Gentz   (Universität Bielefeld)
Friedrich Götze   (Universität Bielefeld)
Ildar Ibragimov   (Academy of Sciences, St. Petersburg)
Tobias Jakobi   (Universität Bielefeld)
Holger Kösters   (Universität Bielefeld)
Franz Lehner   (Technische Universität Graz)
Hans Maassen   (Radboud University, Nijmegen)
Jonathan Novak   (Queen's University)
Mikael de la Salle   (École normale supérieure)
Piotr Śniady   (University of Wroclaw)
Roland Speicher   (Queen's University)
Justine Swierkot   (Universität Bielefeld)
Alexander Tikhomirov   (Komi Research Centre of RAS, Syktyvkar)
Felipe Torres   (Universität Bielefeld)
Martin Venker   (Universität Bielefeld)
Andrei Zaitsev   (Steklov Institute, St. Petersburg)
Dmitry Zaporozhets   (Steklov Institute, St. Petersburg)

(as of 02 December 2009)