



„Workshop on Free Probability and Random Combinatorial Structures“

6 - 8 December 2010

Bielefeld University
Department of Mathematics
V3-201 (Common Room)

This workshop is part of the conference program of the DFG-funded CRC 701
Spectral Structures and Topological Methods in Mathematics
at Bielefeld University

Organizer: Friedrich Götze

Monday, December 6:

9:00 – 10:00 Roland Speicher (Universität des Saarlandes, Saarbrücken)

„Free stochastic analysis, Wigner chaos and the fourth moment“

Abstract: I will recall the basic facts about free Brownian motion and its corresponding stochastic calculus on „Wigner“ space. (No prior knowledge on those topics is assumed.) Then I will report the main result of a recent work with T. Kemp, I. Nourdin, and G. Peccati: We prove that a normalized sequence of multiple Wigner integrals (in a fixed order of free Wigner chaos) converges in law to the standard semicircular distribution if and only if the corresponding sequence of fourth moments converges to 2, the fourth moment of the semicircular law. This extends to the free probabilistic setting some recent results by Nualart and Peccati on characterizations of Central Limit Theorems in a fixed order of Gaussian Wiener chaos.

10:00 – 11:00 Carlos Vargas (Universität des Saarlandes, Saarbrücken)

„Deterministic Equivalents for Random Matrix Models and Operator-Valued Free Probability“

Abstract: In this talk, we present several models of the form

$$f_1^{(N)} := f \left(R_1^{(N)}, \dots, R_p^{(N)}, D_1^{(N)}, \dots, D_q^{(N)} \right), \quad (1)$$

where $R_1^{(N)}, \dots, R_p^{(N)}$ are random matrices, $D_1^{(N)}, \dots, D_q^{(N)}$ are deterministic matrices and f is a self-adjoint function.

Some widely used random matrix ensembles are asymptotically free from deterministic ensembles and converge towards circular, semicircular, or Haar unitary elements in *-probability spaces. Therefore, as the dimensions of the matrices become large, the spectral distribution of $f^{(N)}$ approaches that one of $g := f(r_1, \dots, r_p, d_1, \dots, d_q)$, where the matrices are replaced by the limiting abstract elements. For some choices of f , the distribution of g can be computed explicitly using scalar-valued free probability. In order to treat a wider class of functions, we need to consider the operator-valued generalization of free probability. Then, from the operator-valued Cauchy transform of g one obtains a deterministic equivalent to the Cauchy transform of $f^{(N)}$.

11:00 – 11:30 Coffee break

11:30 – 12:30 Gennadiy Chistyakov (University of Bielefeld)

„Asymptotic Expansions in the CLT in Free Probability“

Abstract: We prove Edgeworth type expansions for free random variables. Our approach relies on the analytic definition of additive free convolution.

12:30 – 14:00 Lunch break

14:00 – 15:00 Marek Bozejko (University of Wrocław)

„New characterisation of free Meixner processes and some applications“

Abstract: We will present generalized stochastic processes with freely (classically) independent values. They have representation as $P(t) = a^*(t) + a(t) + \lambda(t)a^*(t)a(t) + \eta a^*t)a^2(t)$, here $a(t)$ and $a^*(t)$ are free(classical) annihilation and creation distribution and λ and η are continuous functions on non-atomic measure space (T, dx) . If those functions are constant we get the free(classical) representation of exactly Brownian motion, Poisson, gamma, case $\eta = 0$ and Pascal and Meixner processes, when $\eta > 0$. We obtain the free version of results of E. Lytvynov and we get a new characterization of that class of processes. In the second part of talk we give applications of free processes the free version of the Bessis-Moussa-Villani, BMV-conjecture done with G. Chistyakov and F. Götze. Some relations with the papers with Demni on Meixner families will be also done.

15:00 – 16:00 Uwe Franz (Université de Franche-Comté)

„Lévy processes on compact quantum groups and their Markov semigroups“

Abstract: We recall the theory of Lévy processes on compact quantum groups and the classification of their generators (Hunt's formula). Then we present recent work on the extension of Hunt's formula for $SU_q(2)$ by Schürmann and Skeide to the families $SU_q(N)$ and $U_q(N)$. We also discuss properties of the associated Markov semigroups. (Joint work with Anna Kula).

16:00 - 16:30 Coffee break

16:30 – 17:30 Franz Lehner (TU Graz)

„Free Linear and Quadratic Forms“

Abstract: Lukacs' celebrated theorem characterizes the gauss distribution by independence of linear and quadratic forms. We prove a free version of this theorem, assuming only existence of the second moment. Joint work with G.Chistyakov and F.Götze

Tuesday, December 7:

9:00 – 10:00 Anatoly Vershik
(Steklov Institute of Mathematics, St.Petersburg)

„Invariant measures on the lattices of subgroups of noncommutative groups and characters“

Abstract: One of the observations in the theory of noncommutative groups is that some characters of the group are equal to the measure of fixed points for an action of that group. As it becomes clear now this action is the adjoint action on the lattice of the subgroups of the group. From the other side this observation gives a new and very efficient way to construct representations which correspond to that characters. In the talk this program will be illustrated on the example of the infinite symmetric group $\mathfrak{S}_{\mathbb{N}}$.

10:00 – 11:00 Pierre-Loïc Méliot (École Normale Supérieure, Paris)

„Central limit theorems and large deviations for models of random partitions“

Abstract: We shall consider probability measures on the sets of integer partitions coming from the representation theory of the symmetric groups: the Plancherel measures, the Schur-Weyl measures and the Gelfand measures. These three models satisfy the same central limit theorem (namely, Kerov's central limit theorem), up to some minor modifications. The proof of this result relies on the combinatorial properties of a commutative algebra of „observables“ of Young diagrams. If time permits, we shall also discuss large deviations for these models.

11:00 – 11:30 Coffee break

11:30 – 12:30 Kürsat Aker (TUBITAK Feza Gürsey Institute, Istanbul)

„Multiplicative Generators for the Hecke ring of the Gelfand Pair $(S(2n), H(n))$ “

Abstract: For a given positive integer n , Gelfand pair $(S(2n), H(n))$ resembles the symmetric group $S(n)$ in numerous ways. Here, $H(n)$ is a hyperoctahedral subgroup of the symmetric group $S(2n)$. In this talk, we will exhibit a new similarity between the Hecke ring of the pair $(S(2n), H(n))$ and the center of the integral group ring of $S(n)$. Multiplicative generators for centers of integral symmetric groups were first identified by Farahat and Higman, which were later shown to be elementary symmetric polynomials in the celebrated Young-Jucys-Murphy elements by Jucys. In this talk, we will present a set of multiplicative generators for the Hecke ring of $(S(2n), H(n))$, affirming a conjecture of Matsumoto, who showed these elements are elementary symmetric polynomials evaluated at odd Young-Jucys-Murphy elements after projecting from the integral group ring of $S(2n)$ to the Hecke ring of $(S(2n), H(n))$. This is a joint work with Mahir Bilen Can, Tulane University.

12:30 – 14:00 Lunch break

14:00 – 15:00 Gernot Akemann, (Department of Mathematical Sciences Brunel, University West London)

„Non-Hermitian random matrices and orthogonal polynomials in the complex plane“

Abstract: The classical Gaussian random matrix ensembles can be solved in terms of standard Hermite and Laguerre polynomials. In this talk I will show how to extend this statement to their non-Hermitian counterparts. In particular this leads to non-Gaussian weight functions for the respective Laguerre polynomials in the complex plane.

15:00 – 16:00 Alexander Tikhomirov (Komi Research Centre of RAS, Syktyvkar)

„Generalization of the circular law for random matrices“

Abstract: We consider the limit of the empirical spectral distribution of the product of $n \times n$ random matrices $\mathbf{X}^{(1)}, \dots, \mathbf{X}^{(m)}$ with independent entries. Under minimal moment assumptions it is shown that the limit distribution is the distribution with density

$$p(x, y) = \frac{1}{m\pi(x^2 + y^2)^{\frac{m-1}{m}}}.$$

For $m = 1$ we have the well-known circular law. The result was obtained joint with Professor Götze.

16:00 - 16:30 Coffee break

16:30 – 17:30 Holger Kösters (University of Bielefeld)

„Some Observations on Sample Covariance Matrices“

Abstract: The investigation of high-dimensional sample covariance matrices derived from random matrices with independent and identically distributed entries is a familiar topic in random matrix theory. However, for many applications (e.g. in the field of statistics), extensions to more general classes of distributions seem desirable. In this talk I will present some results in this direction, obtained in joint work with Nouredine El Karoui (Berkeley).

**19:00 – Joint dinner at Bültmannshof
Bültmannshof, Kurt-Schumacher-Str. 17a, 33615 Bielefeld**

Please note: For the dinner prior registration is required.

Wednesday, December 8:

**9:00 – 10:00 Leonid Pastur (B.I.Verkin Institute
for Low Temperature Physics and Engineering, Kharkov/Paris)**

„Limiting Laws for Spectral Statistics of Large Random Matrices“

Abstract: We consider certain functions of eigenvalues and eigenvectors (spectral statistics) of real symmetric and hermitian random matrices of large size. We first show that for these functions an analog of the Law of Large Numbers is valid as the size of matrices tends to infinity. We then discuss the scale and the form of limiting fluctuation laws of the statistics and show that the laws can be standard Gaussian (i.e., analogous to usual Central Limit Theorem for appropriately normalized sums of i.i.d. random variables) in non-standard asymptotic settings, certain non Gaussian in seemingly standard asymptotic settings, and other non Gaussian in non-standard asymptotic settings.

**10:00 – 11:00 Ildar Ibragimov (RAS, St.Petersburg)
Dmitry Zaporozhets (Steklov Institute of Mathematics,
St.Petersburg)**

„On Level Sets of Random Gaussian Fields“

Abstract: Let $G(x), x \in \mathbb{R}^d$, be a Gaussian random field. We suppose $G \in \mathcal{C}^1(\mathbb{R}^d)$ with probability one. In this talk we consider some geometric characteristics of level sets of $G(x)$.

11:00 – 11:30 Coffee break

**11:30 – 12:30 Mikhail Gordin (Steklov Institute of Mathematics,
St.Petersburg)**

„Probabilistic extensions and limit theorems“

Abstract: Probabilistic extension can be constructed for a wide class of dynamical system enjoying a kind of expansion or/and contraction in some directions. The extended phase space is, by its construction, a path space of some random process. Hence, it is automatically furnished with a collection of sigma-fields. Lifting the original limit problem to the extended space allows us to avoid any partitioning of the original space (which is a standard way to study dynamical systems probabilistically). Though one needs some additional structures on the original phase space to make this approach applicable, in many situations one obtains stronger results and simpler proofs. From more conceptual point of view this is an example of treating such objects as foliations by means of their „probabilistic unfoldings“.

12:30 – 14:00 Lunch break

14:00 – 15:00 Piotr Sniady (University of Wrocław)

**„Representations of Lie groups and random matrix theory“
(joint work with Benoit Collins)**

Abstract: We study asymptotics of representations of a) a fixed Lie group, b) of the unitary groups $U(n)$ in the limit $n \rightarrow \infty$. We show that in many aspects such representations behave like large random matrices. In particular, in the case b) the limit can be described by free probability and free probability of higher order. Our method of proof is by regarding a representation as a quantum random matrix, i.e. a random matrix with non-commuting entries. This observation gives a heuristic explanation to some well-known connections between random matrix theory and representation theory.

15:00 – 16:00 Maciej Dolega (University of Wrocław)

„Polynomial functions on the set of Young diagrams and colorings of bipartite graphs“

Abstract: Polynomial functions (in the sense of Kerov and Olshanski) on the set of Young diagrams are functions which have a prominent role in the asymptotic representation theory of a permutation groups. We will show how to construct a function on the set of Young diagrams from a given bipartite graph and when these kind of functions are polynomial functions. We will give a connection of our result with Jack symmetric functions and some conjectures of Lassalle. Our method involves a differential calculus on the set of Young diagrams and combinatorics of bipartite graphs which will be also discussed.

This is a joint work with Piotr Sniady.

16:00 Closing of the workshop and coffee