RESEARCH AGENDA

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Research Interests

The main characteristic of my research work is that I bring together methods and ideas from the theories of Topological Transformation Groups, Dynamical Systems, Actions of Algebraic and Lie Groups, Dynamics of Linear Operators and Operator Theory in order to study the dynamic behavior of various objects in these theories.

My Research Agenda contains two main topics. Namely

⋆ **Linear Dynamics on finite and infinite dimensional vector spaces**

⋆ **Topological Transformation Groups, Isometric Group Actions and Dynamical Invariants in Compactifications**

These research topics share common ideas, problems and even common methods when possible. A general characteristic is the use of asymptotic methods and various concepts of limit sets coming from the stability theory of Dynamical Systems. For more details please take into consideration the brief analysis of my publications.

⋆ Ideas and concepts from Topological Transformation Groups and Dynamical Systems in the study of Dynamics of Linear Operators on finite and infinite dimensional vector spaces

It was back in the summer of 2005 when I met George Costakis, a specialist in Linear Dynamics. He mentioned to me the phenomenon of *Hypercyclicity* in Linear Dynamics and he asked me if we can apply some of the methods or use concepts from my previous research work, which was the study of special group actions like the natural action of the group of isometries on a locally compact metric space or the study of proper and properly discontinuous actions on locally compact spaces. The concept of Hypercyclicity has to do with the action of the semigroup of the iterates T^n, $n \in \mathbb{N}$ of a bounded linear operator T on a Banach or Hilbert space X: An operator $T : X \to X$ is called hypercyclic if it has a dense orbit, i.e. there is a vector $x \in X$ such that its orbit $\mathrm{Orb}(T, x) := \{T^n x, n = 0, 1, \ldots\}$ is dense in X. My question was if something like that can appear in linear dynamics and if yes what kind of operators can have this dynamical behavior. The answer was that many examples of well known operators are hypercyclic and moreover they have a dense set of periodic points and a G_δ set of point with dense orbits, they are chaotic! For the case of a Hilbert space, the set of hypercyclic operators is dense in the space of all bounded linear operators with respect to the strong operator topology, i.e. with respect to the pointwise convergence. To mention only some concrete examples of chaotic operators:

(i) The translation operator $T_\alpha : H(\mathbb{C}) \to H(\mathbb{C})$ on the space $H(\mathbb{C})$ of all entire functions on $\mathbb{C}$ endowed with the topology of uniform convergence on compact sets, defined by $T_\alpha(f) = f(z+\alpha)$, where $z \in \mathbb{C}$ and α is a non-zero complex number (G. D. Birkhoff 1929).
(ii) The derivative operator $D(f) = f'$ on $H(\mathbb{C})$ (G. R. MacLane 1951).
(iii) For every scalar λ of modulus greater than 1 the operator λB on $l^2(\mathbb{N})$ where B is the backward shift (S. Rolewicz 1969).

It is somewhat surprising that hypercyclic operators exist since they do not exist on finite-dimensional Banach spaces. The concept of hypercyclicity is closely related to the invariant subset problem like the concept of cyclicity is related to the invariant closed subspace problem. It is an easy observation that an operator lacks non-trivial invariant subsets if and only if every non-zero vector has a dense orbit. For an account of results on hypercyclicity we refer to the recent book of F. Bayart - É. Matheron [6] and K.-G. Grosse-Erdmann - A. Peris [14].

This first characterization of hypercyclicity comes from topological dynamics and it is known as the Birkhoff's transitivity theorem: Let T be a bounded linear operator on a separable Banach space X. Then T is hypercyclic if and only if it is topologically transitive; that is, for every pair of non-empty open sets U, V of X there exists a positive integer n such that $T^n U \cap V \neq \emptyset$. Birkhoff's transitivity theorem can be reformulated using concepts from the theories of Dynamical Systems and Topological Transformation Groups, namely the concept of the limit set:

$$L(x) = \{y \in X : \text{ there exists a strictly increasing sequence of positive integers } \{k_n\} \text{ such that } T^{k_n} x \to y\}$$

that describes the limit behavior of an orbit and the generalized (prolongational) limit set that describes the asymptotic behavior of the orbits of nearby points to $x \in X$:

$$J(x) = \{y \in X : \text{ there exist a strictly increasing sequence of positive integers } \{k_n\} \text{ and a sequence } \{x_n\} \subset X \text{ such that } x_n \to x \text{ and } T^{k_n} x_n \to y\}.$$

Limit and extended limit sets have their roots in the Qualitative Theory of Dynamical Systems when they are used mainly to describe the Lyapunov and the asymptotic stability of an equilibrium point or, more generally, of a compact minimal set. They, also, "encode" information which allows us to connect the global structure of the underlying space with local properties, like we do with proper and properly discontinuous actions for example where the limit and the extended sets are empty. In view of the concept of limit sets a bounded linear operator $T : X \to X$ on a separable Banach space is hypercyclic if $L(x) = X$ for some non-zero vector $x \in X$. And Birkhoff's transitivity theorem says that T is hypercyclic if and only if $J(x) = X$ for every $x \in X$. Our first idea with G. Costakis was to "localize" the concept of topological transitivity using J-sets. In [7], we proposed a new class of operators, called J-class operators by requiring the existence of a non-zero vector $x \in X$ such that $J(x) = X$. There are many examples of operators such that these two concepts coincide but there are also examples that they do not since J-class operators may occur also on non-separable linear spaces as we showed in [7].

As a new concept J-class operators and J-sets had a satisfactory acceptance by the mathematical community: In a recent books of F. Bayart - É. Matheron *Dynamics of linear operators*, Cambridge University Press [6] and K.-G. Grosse-Erdmann - A. Peris *Linear Chaos*, Universitext, Springer [14], which are actually the first published books concerning dynamics of linear operators, they referred to our work and they used J-sets and our asymptotic technics from [7]. There is also a Ph.D. in progress by A. Bahman Nasseri in the Technische Universität of Dortmund under the supervision of Prof. Dr. Rainer Brück with subject the study and the existence of J-class operators on certain Banach spaces and also a series of paper by M.R. Azimi and V. Müller, G. Tian and B. Hou, K. Chan and I. Seceleanu and A. Bahman Nasseri that study or use J-class operators.

An example how ideas and concepts and methods from Topological Transformation Groups and Dynamical Systems can be applied in the study of Dynamics of Linear Operators on finite and infinite dimensional vector spaces is given in [16] in which paper we provided a "tool" for studying topologically transitive operators on non-separable F-spaces (i.e. complete and metrizable vector spaces) by using technics and already known results from the theory of hypercyclic operators.

Future Research Plan: Our research plan is to study the concept of a J-class operator in a variety of classes of operators like unilateral or bilateral weighted shifts on $l^p(\mathbb{N})$, $l^p(\mathbb{Z})$ $l^\infty(\mathbb{N})$ and $l^\infty(\mathbb{Z})$ and to study the concept of a J-class operator in relation with perturbations of well known operators like the shifts or the Volterra operator. We plan to do the same after introducing *two new concepts*:

- The first concept is the concept of a *recurrent operator*: A bounded linear operator $T : X \to X$ will be called recurrent if the set of vectors which are recurrent, i.e. $x \in L(x)$, is dense in X or, equivalently, $x \in J(x)$ for every $x \in X$.
- The second concept is the concept of a *zero J-class operator*: A bounded linear operator $T : X \to X$ will be called zero J-class if the only vector such that $J(x) = X$ is the zero vector.

Related papers: [7], [16], [12], [8], [11], and [15].

There are remarkably many similarities between the dynamical behavior of a finitely generated abelian linear algebraic or a Lie group and the dynamics of the semigroup generated by a single bounded operator on a separable infinite dimensional Banach space. Bringing together methods, ideas and concepts from both of these theories we studied together with H. Abels several problems concerning these two classes of dynamics. As an example we mention the joint papers with H. Abels [3], [1] in which the original problems came from the theory of linear operators on infinite dimensional spaces. In these papers we brought together results about the density of subsemigroups of abelian Lie groups, the minimal number of topological generators of abelian Lie groups and a result about actions of algebraic groups. On the other hand, in [9] and [10], we worked with G. Costakis and D. Hadjiloucas on several problems related to J-class or hypercyclic tuples of matrices. The methods we used in these papers are of combinatorial nature.

Future Research Plan: We plan to give a complete description of the dynamics of a finitely generated abelian linear algebraic group in terms of the J-sets.
Related papers: [3], [1], [9] and [10].

⋆ Topological Transformation Groups, Isometric Group Actions and Dynamical Invariants in Compactifications

Future Research Plan: Continuing older research topics, we work with H. Abels and P. Strantzalos on a series of "old fashioned" problems on Topological Transformation Groups and Isometric Group actions which have strong connections with current research trends in Mathematics, for instance our joint work with H. Abels and G. Noskov "Proper actions and proper invariant metrics" [4] used in a strong way in the paper of P. Müller and C. Richards "Ergodic properties of randomly coloured point sets".

Group of isometries of locally compact metric spaces play a key role in our research as also embeddings of proper actions in appropriate zero-dimensional compactifications in order to study dynamical invariants of the original group action.

Related papers: [2], [4], [18], [19], [20], [21] and [22].

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Closing comment: *As a closing comment we would like to point out that all the research plans are based on concepts related to limit and extended limit sets which are in the heart of all the problems we investigate. Limit and extended limit sets do not form a theory but they are elegant and efficient "tools" that encode certain dynamical information.*

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