Homework
Waves in Evolution Equations
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Due: Wed. April 26, 12:00, V3-128, mailbox 128 (Christian Döding)

Tutorial: Tue. 02.05. 2017, 14-16, V5-148

Exercise 1: [The Nagumo equation]

(a) Verify that the Nagumo equation

$$u_t = u_{xx} + u(1-u)(u-b), \quad x \in \mathbb{R}, t \geq 0 \tag{1}$$

with $0 < b < 1$ has a travelling wave solution with profile $v_\star$ and speed $c_\star$ given by

$$v_\star(\xi) = \frac{1}{1+\exp(-\frac{\xi}{\sqrt{2}})}, \quad c_\star = \sqrt{2}\Big(b - \frac{1}{2}\Big). \tag{2}$$

(b) Show that the general Nagumo equation

$$u_t = Du_{xx} + B(u-b_1)(b_2-u)(u-b_3), \quad x \in \mathbb{R}, t \geq 0 \tag{3}$$

with constants $D, B > 0$ and $b_1 < b_2 < b_3$ can be transformed into the standard form (1) by transformations of the type

$$u(x,t) = \beta_1 v(x,t) + \beta_2, \quad v(x,t) = w(\alpha_1 x, \alpha_2 t), \quad x \in \mathbb{R}, t \geq 0 \tag{4}$$

with suitable constants $\beta_1, \beta_2, \alpha_1, \alpha_2 \in \mathbb{R}$.

(c) Determine the profile and the speed of a travelling wave for the general Nagumo equation (3).

(3+6+3 points)