Homework
Waves in Evolution Equations
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Due: Wed. May 03, 12:00, V3-128, mailbox 128 (Christian Döding)

Tutorial: Tue. 09.05. 2017, 14-16, V5-148

Exercise 2: [The quintic Nagumo equation]

(a) Show that the quintic Nagumo equation

$$u_t = Du_{xx} - B\prod_{j=1}^{5}(u-\beta_j), \quad x \in \mathbb{R}, t \geq 0 \tag{1}$$

with parameters $\beta_1 < \beta_2 < \beta_3 < \beta_4 < \beta_5$ and $D, B > 0$ can be brought by suitable transformations into the normal form

$$u_t = u_{xx} + f(u), \quad f(u) = u(u-b_2)(u-b_3)(u-b_4)(1-u), \tag{2}$$

where $0 < b_2 < b_3 < b_4 < 1$.

(b) Set up the travelling wave ODE (called TWODE(c)) that determines travelling waves with speed c for the equation (2). Find a relation between the zeroes of f that allows to determine an orbit connecting 0 to b_3 and b_3 to 1, respectively, by solving a first order scalar autonomous differential equation.

(c) Compute all steady states of TWODE(c) and determine whether they are sources, sinks or saddles. Use NUMLAB to draw the phase portrait of TWODE(c) for some selected values of b_2, b_3, b_4 and c. Draw the stable and unstable manifolds of the saddles. For each of the cases $0 \to b_3$, $b_3 \to 1$ and $0 \to 1$ find at least one parameter setting where these connections occur.

(3+3+4 points)

Exercise 3: [The Klein Gordon equation] Write the Klein-Gordon equation

$$u_{tt} = u_{xx} - u, \quad x \in \mathbb{R}, t \geq 0 \tag{3}$$

as a first order system

$$U_t + AU_x + BU = 0, \quad A, B \in \mathbb{R}^{2,2}, x \in \mathbb{R}, t \geq 0, \tag{4}$$

by setting $U = (u, u_t - u_x)^\top$. Transform also the initial data $u(\cdot, 0) = u_0, u_t(\cdot, 0) = v_0$, with given functions $u_0, v_0 : \mathbb{R} \to \mathbb{R}$ into initial data for the system. Determine the eigenvalues and eigenvectors of A. Show that wave solutions of (3) transform into wave solutions of (4) and vice versa.

(6 points)