Homework
Waves in Evolution Equations
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Due: Wed. May 10, 12:00, V3-128, mailbox 128 (Christian Döding)

Tutorial: Tue. 16.05. 2017, 14-16, V5-148

Exercise 4: [The nonlinear Schrödinger equation]
Consider the nonlinear Schrödinger equation

$$u_t = iu_{xx} + ib|u|^2u,\ x \in \mathbb{R}, t \geq 0, \quad b > 0, \quad u(x,t) \in \mathbb{C}, \tag{1}$$

and determine a wave solution which travels and rotates simultaneously. Use the ansatz

$$\begin{aligned} u(x,t) =& \exp(-i\theta t)v(x-ct), \quad x,t \in \mathbb{R} \\ v(\xi) =& c_1 \exp(ic_2\xi)\mathrm{sech}(c_3\xi) \end{aligned}$$

with suitable constants θ, c, c_1, c_2, c_3 (recall $\mathrm{sech}(x) := \frac{1}{\cosh(x)}, x \in \mathbb{R}$). For some interesting parameter set use MATLAB (or some other package) to draw the graph of $(x,t) \to \mathrm{Re}(u)(x,t)$ over a suitable domain $[x_-, x_+] \times [0,T]$.

(8 points)

Exercise 5: [Travelling wave of a damped nonlinear wave equation]
Let $u(x,t) = v_\star(x-ct)$ be a travelling wave solution of the scalar parabolic equation

$$u_t = Au_{xx} + f(u), \quad x \in \mathbb{R}, t \geq 0, \tag{2}$$

where $A > 0$ and $f \in C^1(\mathbb{R}, \mathbb{R})$. Find a travelling wave of the damped wave equation

$$Mv_{tt} + v_t = Av_{xx} + f(v), \quad x \in \mathbb{R}, t \geq 0, \tag{3}$$

with a given $M > 0$. Use the ansatz $v_\star(k\xi), \xi \in \mathbb{R}$ with a suitable k for the profile and determine a suitable speed. How do the speed and the profile behave as $M \to 0$?

(6 points)