Homework
Waves in Evolution Equations
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Due: Wed. June 7, 12:00, V3-128, mailbox 128 (Christian Döding)

Tutorial: Tue. 13.06 2017, 14-16, V5-148

Exercise 12: [Travelling waves of a viscous conservation law II]

Consider the parabolic equation from Exercise 11

$$u_t = u_{xx} - [f(u)]_x, \quad x \in \mathbb{R}, \quad f \in C^1(\mathbb{R}, \mathbb{R}) \tag{1}$$

and let the assumptions (a) and (b) from Exercise 11 hold. Let $v_\star$ be the profile and $\mu_\star$ be the speed of the travelling wave satisfying $\lim_{\xi \to \pm\infty} v_\star(\xi) = v_\pm$. The travelling wave solves the equation

$$0 = v_{xx} + \mu_\star v_x - [f(v)]_x, \quad x \in \mathbb{R}, t \geq 0. \tag{2}$$

(a) Show that the endpoints $w_\pm = (v_\pm, 0)$ are not saddles of the equation (2) when written as a first order system.

(b) Calculate the travelling wave above explicitly for two given values $v_+ < v_-$ in case of Burgers equation

$$u_t = u_{xx} - \frac{1}{2}[u^2]_x, \quad x \in \mathbb{R}, \quad f \in C^1(\mathbb{R}, \mathbb{R}) \tag{3}$$

(7 points)

Exercise 13: [Abstract framwork for oscillating waves]
Show that the assumptions (A1)-(A4) of the abstract framework (Section 3.6) are satisfied for the following equation

$$u_t = a u_{xx} + g(|u|)u, \quad x \in \mathbb{R}, t \geq 0, u(x,t) \in \mathbb{C} \tag{4}$$

where $a \in \mathbb{C}$, $g \in C^1(\mathbb{R}, \mathbb{C})$ with the following settings

$$X = L^2(\mathbb{R}, \mathbb{C}), Y = H^2(\mathbb{R}, \mathbb{C}), F(v) = v_{xx} + g(v)v, \quad \text{for } v \in X \tag{5}$$
$$G = S^1 = \mathbb{R}/_{2\pi\mathbb{Z}}, (a(\gamma)v)(x) = \exp(i\gamma)v(x), x \in \mathbb{R}, \gamma \in G, v \in X. \tag{6}$$

Calculate the derivative $d_\gamma[a(\mathbb{1})v]$ for $v \in Y$.

(7 points)