Dr. Kilian Raschel
Daniel Altemeier
Department of Mathematics
Bielefeld University

SS 2011

Stochastik 2 - Exercises 1

Handover date: **Friday, Apr 15th, 10:00**

Please put your solutions into the mailbox 200 which belongs to the head of the tutorials, Ms. Katharina von der Lühe. The mailbox can be found in the copy-room V3-128. Before the insertion of the solution please check that the sheets are ordered correctly and tacked. Write down your name in a legible handwriting on the the first sheet of your solution.

Exercise 1.I:
Consider $\big(Y_n\big)_{n\in\mathbb{N}}$ to be i.i.d. equally distributed random variables on the probability space $\big(\Omega,\mathbb{P}\big)^{\otimes\infty} = \big(\Omega^\infty,\mathbb{P}^{\otimes\infty}\big)$ with $Y\big(\Omega\big) = \{1,2\}$. Define the (random) process $X = \big(X_n\big)_{n\in\mathbb{N}}$ by

$$X_n(\omega) := 2Y_n(\omega) + Y_{n+1}(\omega) \quad \text{for all } \omega \in \Omega^\infty \text{ and } n \in \mathbb{N}.$$

Show, that X is a Markovian Chain and specify the transition probabilities of X.

Exercise 1.II: (maximum of the symmetric random walk)
Let $\big(Y_n\big)_{n\in\mathbb{N}}$ i.i.d. random variables on some probability space $\big(\Omega,\mathbb{P}\big)$, equally distributed on $\{-1,1\}$ and consider the partial sums

$$S_n(\omega) := \sum_{i=1}^{n} Y_i(\omega) \text{ for } n \in \mathbb{N}.$$

(Note, that this $(S_n)_{n\in\mathbb{N}}$ is indeed the classical case of symmetric random walk)
Now, denote by $\big(X_n\big)_{n\in\mathbb{N}}$ the *running maximum* of $\big(S_n\big)_{n\in\mathbb{N}}$, i.e.

$$X_n := \max\big\{\ S_i \bigm| i \le n\ \big\} \text{ for } n \in \mathbb{N}.$$

Is $X = \big(X_n\big)_{n\in\mathbb{N}}$ a Markovian Chain? Prove your answer.

Exercise 1.III:

The transition probabilities of a Markovian Chain with state space $\{S_1, \dots, S_7\}$ are represented by the following stochastic matrix

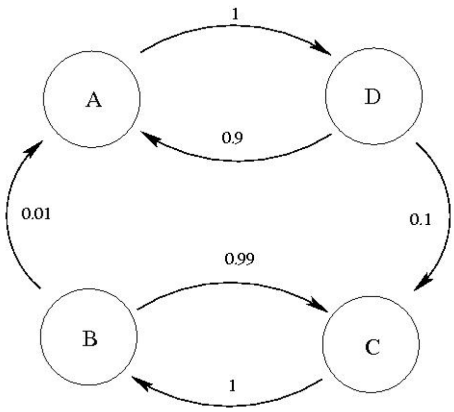
$$P = \begin{pmatrix} 0 & 0 & \frac{1}{3} & 0 & \frac{1}{2} & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{2}{5} & 0 & 0 & \frac{3}{5} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & \frac{1}{4} & 0 & \frac{3}{4} & 0 & 0 & 0 \end{pmatrix}$$

(a) Draw the transition graph, i.e. a graph for which there is a 1-1-correspondence between nodes of the graphs and states of the Markovian Chain. Furthermore draw in directed edges between nodes whereever there is positive probability for a transition between the corresponding states. Also write down the transition probabilities at the edges.

(b) Let the transition probabilities of a process X be given by the stochastic matrix P, and assume that $X_0 = \delta_{S_1}(\cdot)$ for all $\omega \in \Omega$. Give the probability distribution of the random variable X_2, use a computer to figure out the distributions of X_{10}, X_{100} and X_{1000}.

Exercise 1.IV:

Figure 1:



Find the stochastic matrix that corresponds to the above transition graph and give the distribution of X_1, if we assume that X_0 is the equal distribution on $\{A, B, C, D\}$.
Again use your computer to find the (approximative) distributions of X_{10}, X_{100} and X_{1000}.