Dr. Kilian Raschel
Daniel Altemeier
Department of Mathematics
Bielefeld University

SS 2011

Stochastik 2 - Exercises 10[1]

Handover date: **Friday, June 24th, 10:00**

Please put your solutions into the mailbox 200 which belongs to the head of the tutorials, Ms. Katharina von der Lühe. The mailbox can be found in the copy-room V3-128. Before the insertion of the solution please check that the sheets are ordered correctly and tacked. Write down your name in a legible handwriting on the the first sheet of your solution.

Exercise 10.I:

Remember **Exercise 9.III** where we computed $p_{23}(\cdot)$ of the Q-matrix

$$Q = \begin{pmatrix} -2 & 1 & 1 \\ 4 & -4 & 0 \\ 2 & 1 & -3 \end{pmatrix}.$$

1. Gather $p_{23}(t)$ (you may use your results from exercise 9.III),
2. Find an invariant distribution λ for the Q-matrix,
3. Show that $\lim_{t \nearrow \infty} p_{23}(t) = \lambda_3$.

Exercise 10.II:

Consider the below matrices as Q-matrices

$$\text{(i)} \quad A = \begin{pmatrix} -2 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 1 & 0 & 0 & -1 \end{pmatrix} \qquad \text{(ii)} \quad B = \begin{pmatrix} -1 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & 0 & -2 & 2 \\ 0 & 0 & 2 & -2 \end{pmatrix}$$

1. Draw the transition rate graph,
2. Compute $\lim_{t \to \infty} \mathbb{P}\big[X_t = 2 \big| X_0 = 1\big]$ and $\mathbb{P}\big[X_t = 3 \big| X_0 = 1\big]$ for both matrices.
3. Find all the invariant distribution for each of the Q-matrices. What is apparent?

[1] As the next thursday June 23rd is (again) a feast day and as the remaining time until the written exam (July 11th) is getting short, there will be a (last) exercise-sheet 11 available online :

www.math.uni-bielefeld.de/∼daltemeier/sto2-ex11.pdf

It will be uploaded at last on thursday 23rd until 18:00.

Exercise 10.III:

Consider a fleet of N buses. Each bus breaks down independently at rate μ, when it is sent to the depot for repair. The repair shop can only repair one bus at a time and each bus takes an exponential time of parameter λ to repair. Find the invariant distribution of the number of buses in service.