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SS 2011

Stochastik 2 - Exercises 11

Handover date: **Friday, July 1st, 10:00**

Please put your solutions into the mailbox 200 which belongs to the head of the tutorials, Ms. Katharina von der Lühe. The mailbox can be found in the copy-room V3-128. Before the insertion of the solution please check that the sheets are ordered correctly and tacked. Write down your name in a legible handwriting on the the first sheet of your solution.

Exercise 11.I:

Remember **Exercises 8.I** and **8.II**. Use **theorem B.V.1** (backward equation) to show that **Exercise 8.1** and **8.II** actually give a precise instruction how to gain the transition matrix of the jump-matrix corresponding to a given Q-matrix (under proper assumptions). Denote all the assumptions you need on Q to make use of the theorem.

Exercise 11.II:

Use the above gained connection between Q-matrices and the transition matrix of the corresponding jump-matrix to characterize the CTMC with Q-matrix

$$Q = \begin{pmatrix} -4 & 2 & 1 & 1 \\ 0 & -1 & 1 & 0 \\ 3 & 0 & -5 & 2 \\ 1/10 & 0 & 0 & -1/10 \end{pmatrix}$$

Now, we can easily characterize the CTMC, in particular...

1. ... denote the transition matrix of the jump-matrix
2. ... give the estimated residence time for all states $x \in \{1, 2, 3, 4\}$

   $$\mathbb{E}_x \left[\inf_{t \geq 0} \{ M_t \neq x \} \right]$$

3. ... draw "typical path" of the corresponding CTMC,
   Draw a path that visits every state (that has positive probability to be visited) at least once and remains roughly the expected residence time in every state
4. ... compute an invariant distribution (both for the transition matrix of the jump process and the Q-matrix Q),
5. ... for both cases: is the invariant distribution the limit distribution? (Why are we not surprised to see they do not coincide?)