Dr. Kilian Raschel
Daniel Altemeier
Department of Mathematics
Bielefeld University

SS 2011

# Stochastik 2 - Exercises 6

Handover date: **Friday, May 20th, 10:00**

**Please put your solutions into the mailbox 200 which belongs to the head of the tutorials, Ms. Katharina von der Lühe. The mailbox can be found in the copy-room V3-128. Before the insertion of the solution please check that the sheets are ordered correctly and tacked. Write down your name in a legible handwriting on the the first sheet of your solution.**

**Exercise 6.I:**

Let $X = \big(X_n\big)_{n\in\mathbb{N}}$ and $Y = \big(Y_n\big)_{n\in\mathbb{N}}$ be aperiodic Markovian Chains on some probability space $\big(\Omega, \mathbb{P}\big)$, assume that for both the state-space is some countably finite $I$. Further assume that they are both irreducible.
Show:

$$\mathbb{P}\big[\ \exists n \text{ mit } X_n = Y_n\ \big] = 1.$$

**Exercise 6.II:**

Consider a randomly moving particle on some square generated by the points $A, B, C, D$ (Think of $A = (0,0), B = (0,1), C = (1,1)$ and $D = (1,0)$). Per unit of time our particle jumps to one of the neighbouring points, which to one with which the current state has a common edge.

(a) Show that in the symmetric case (particle runs as likely clockwise as counterclockwise), there is an invariant distribution and the Markov Chain is reversible.

(b) Which results remain in the asymmetrical case? Here, do only consider the case, where the particle prefers moving clockwise with 'intensity' 3/4 to 1/4, i.e. standing in some point, there are two attainaible (with positive probability) edges (neighbours) that are in question to be the particle's next position. Each one of them is reached by a clockwise movement or counterclockwise movement, we assume the particle to always and independently move clockwisely with probability $\frac{3}{4}$ and the counterclockwise movement is chosen with probability $\frac{1}{4}$.

**Exercise 6.III:**

Let $P$ be a stochastic matrix on a finite set $I$ and denote by $|I|$ the *cardinality* of the set $I$, i.e. the number of (distinguishable) elements in $I$. Show that a distribution $\pi$ is invariant for $P$ if and only if $\pi(I - P + A) = a$ where $A$ is a $|I| \times |I|$-matrix with '1' in every entry and $a = (1, \ldots, 1)$ of length $|I|$. Show further that if $P$ is irreducible, then $I - P + A$ is invertible.
*(Note that this enables us to compute invariant distributions by inverting a matrix)*