Dr. Kilian Raschel
Daniel Altemeier
Department of Mathematics
Bielefeld University

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Stochastik 2 - Exercises 7 *2nd edition*

Handover date: **Friday, May 27th, 10:00**

Please put your solutions into the mailbox 200 which belongs to the head of the tutorials, Ms. Katharina von der Lühe. The mailbox can be found in the copy-room V3-128. Before the insertion of the solution please check that the sheets are ordered correctly and tacked. Write down your name in a legible handwriting on the the first sheet of your solution.

Exercise 7.I:

Show *Proposition B.1.9*:
If $X = \big(X_t\big)_{t\in[0,\infty)}$ and $Y = \big(Y_t\big)_{t\in[0,\infty)}$ are two independent Poisson-processes, say X with parameter λ and Y with parameter μ. Then the process $Z = \big(X_t + Y_t\big)_{t\in[0,\infty)}$ is also a Poisson-process with parameter $\lambda + \mu$.

Exercise 7.II:

Consider $X = \big(X_t\big)_{t\in[0,\infty)}$ to be a Poisson-process with parameter λ. For arbitrary $n \in \mathbb{N}$, compute the distribution of the random variable

$$S_n := \inf\{t : X_t = n\}.$$

Exercise 7.III:

The number $(X_t)_{t\in[0,\infty)}$ of cars arriving at a gas-station in the time-interval $[0,t]$ is assumed to be a Poisson-process with Parameter λ. Whether or not a car is fueled with diesel, we want to consider as a sequence of independent Bernoulli-trials where $p \in (0,1)$ denotes the probability for each car to be fueled with diesel.

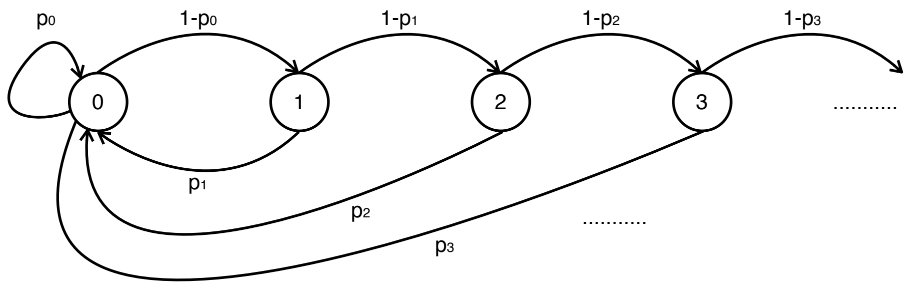
(a) Show that the number $Z = \big(Z_t\big)_{t\in[0,\infty)}$ of 'diesel'-cars in the time-intervals $[0,t]$ forms again a Poisson process with parameter $p\lambda$.

(b) Show that Z and $X - Z$ are independent.

Exercise 7.IV:(One more DTMC)

For a game in each point in time $n \in \{1, 2, \ldots\}$ a certain coin (not necessarily fair) is tossed, assume the coin in time i shows 'head' with probability $p_i \in [0, 1], i = 1, 2, \ldots$. If the coin tossed in time i shows tail, then the player advances on a sequence of states one step upwards, if not, i.e. if the coin shows 'head', then the player returns to state 0. This is illustrated by the transition graph below.

Figure 1:



(a) Show that under the condition $p_i \in (0, 1) \forall\, i \in S = \mathbb{N} \cup \{0\}$ the Markov Chain is irreducible.

(b) Compute the probability of returning to state 0 and the expected time to the first return to 0. Is 0 nullrecurrent or positive recurrent?

(c) With the help of the above result give a relation between the values of p_i and the kind of recurrence of state 0.[1]

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[1] Advice:

You may (without verification) use the classical result:

Let $(a_n)_{n \geq 1}$ be a sequence of values from $(0, 1]$, then the following holds:

$$\sum_{n=1}^{\infty} a_n < \infty \Rightarrow \lim_{m \to \infty} \prod_{n=1}^{m} (1 - a_n) > 0$$

$$\sum_{n=1}^{\infty} a_n = \infty \Rightarrow \lim_{m \to \infty} \prod_{n=1}^{m} (1 - a_n) = 0.$$