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Stochastik 2 - Presence-exercises 1:

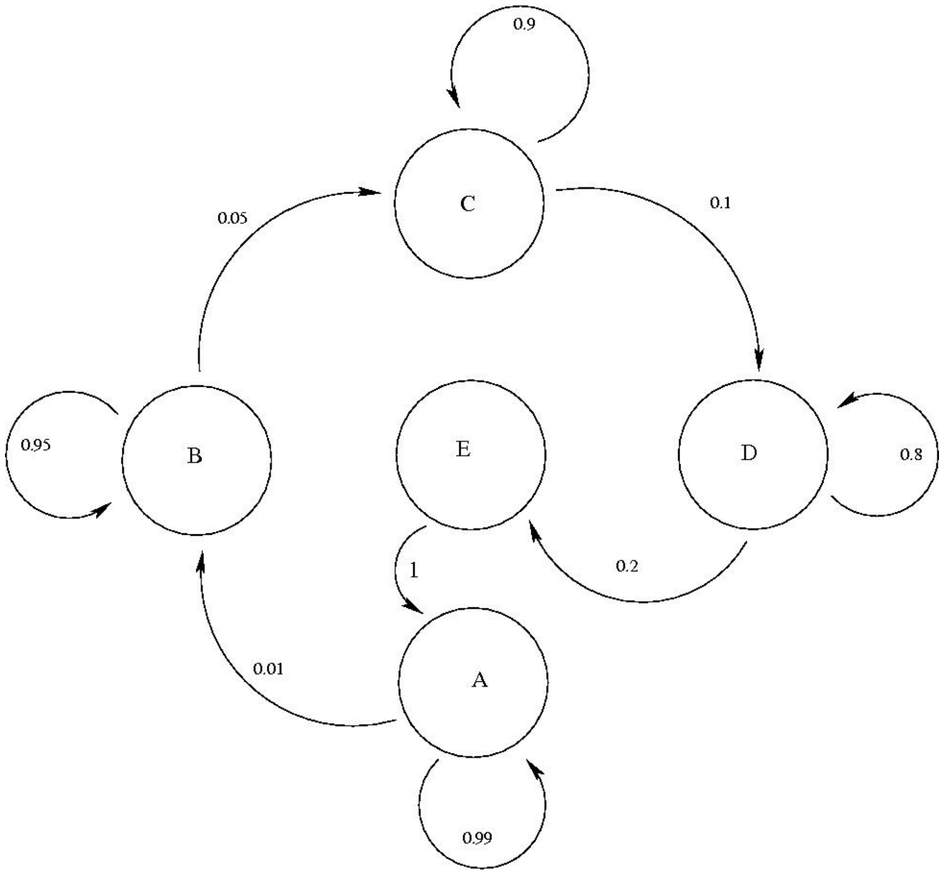
Presence-Exercise 1.I:
An urn contains $m \geq 2$ balls that are consecutively numbered. One progressively draws (randomly with identical probability for each of the balls) a single ball, notes the label and returns it into the urn. Let $X = \big(X_n\big)_{n\in\mathbb{N}}$ denote the number of different balls observed until the n-th drawing.
Is X a Markovian Chain? If that is the case, specify the transition probabilities.

Presence-Exercise 1.II:
For the below stochastic matrix P give the stochastic process $X = (X_n)_{n\in\mathbb{N}}$ that 'evolves according to P', assume therefore, that the state-space is given by $\{S_1, \dots, S_8\}$. Furthermore draw the transition graph of X.

$$P = \begin{pmatrix} 1/2 & 1/4 & 0 & 1/4 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/4 & 3/4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1/2 & 0 & 0 & 1/2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1/3 & 1/3 & 1/3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/3 & 1/3 & 1/3 \\ 0 & 0 & 0 & 0 & 0 & 1/10 & 4/5 & 1/10 \\ 0 & 0 & 0 & 0 & 1/3 & 2/3 & 0 & 0 \end{pmatrix}$$

Presence-Exercise 1.III:

Figure 1:



Find the stochastic matrix that corresponds to the above transition graph and give the distribution of X_1, if we assume that X_0 is the equal distribution.

Presence-Exersice 1.IV:

Are the following claims correct, are they false or are they possible but in general not correct?

- A stochastic matrix P, that is not the unit matrix, can be idempotent, which means:[1] $\exists k \in \mathbb{N} : \quad P^n = I_n$. (Find an example or prove the contrary)
- A stochastic matrix P can be nilpotent, i.e. $\exists k \in \mathbb{N} s.t. \quad P^k = 0_n$.
- The product of two stochastic matrices is again a stochastic matrix, give a proof or a counterexample

[1] I_n denotes the unit-matrix in $\mathbb{R}^n$ and 0_n denotes the zero-matrix in $\mathbb{R}^n$.