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Stochastik 2 - Presence-Exercises 2:

Presence-Exercise 2.I:
Consider the *symmetric random walk with absorbing boundary* on the space $\{0,\ldots,3\}$ where 0 and 3 are the absorbing states.

(a) Find the corresponding transition matrix

(b) Compute the probability for an absorbtion in state 0, if we assume $X_0 = 2$.

Presence-Exercise 2.II:
Consider once more the Markov-Chain from Exercise 1.III:
$$P = \begin{pmatrix} 0 & 0 & \frac{1}{3} & 0 & \frac{1}{2} & \frac{1}{6} & 0 \\ 0 & 0 & 0 & \frac{2}{5} & 0 & 0 & \frac{3}{5} \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2} & 0 & \frac{1}{2} & 0 \\ 0 & 0 & \frac{2}{3} & 0 & 0 & 0 & \frac{1}{3} \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & \frac{1}{4} & 0 & \frac{3}{4} & 0 & 0 & 0 \end{pmatrix},$$
Give all closed classes and decide for all states $\{S_1,\ldots S_7\}$ whether they are recurrent or transient. Prove your claims.
(It may be helpful to observe the transition graph)

Presence-Exercise 2.III:
Study example A.II.1 (Two-state Markov-Chain) to solve the below exercise.

Every morning Tom walks to his office (by foot) and in the evening he returns in the same way. In the case it is raining, he takes his umbrella with him. The problem is of course, that this acting invokes the danger of having the umbrella in the wrong place. Let the appearance of raining in the morning and in the evening and on every day be i.i.d. random variables with the probability p that rain occurs.

(a) Compute the probability that Tom and his umbrella are in different places at time n, if we start on monday's early morning with Tom and his umbrella at home.

(b) Show that having Tom and his umbrella in different places is a recurrent state.