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Stochastik 2 - Presence-Exercises 3:

Presence-Exercise 3.I: (A concrete filtration)

Denote by $\mathcal{P}$ the equal distribution on $\{-1,1\}$, define then $\big(\Omega,\mathbb{P}\big)=\big(\{-1,1\}^{\infty},\mathcal{P}^{\otimes\infty}\big)$. Now denote $X_n=\sum_{i=1}^{n}\omega_i, n\in\mathbb{N}$, set $X_0=0$. Let $\mathcal{F}$ be the natural filtration of the process X on Ω. Denote $\mathcal{F}_0$, $\mathcal{F}_1,\mathcal{F}_2,\mathcal{F}_3$ explicitly by writing down all the sets $\{X_i=k_i, i\leq j\}, 0\leq j\leq 3\ \forall\, k_i\in I=\mathbb{N}$.

To obtain the sets $\mathcal{F}_0,\mathcal{F}_1,\mathcal{F}_2,\mathcal{F}_3$, the above sets must be expanded by all countable unions, intersections and complementation within these sets. So the denoted sets are nor really the sets of $\mathcal{F}_i, 0\leq i\leq 3$.

For property that one receives the sets $\mathcal{F}_i, i\leq 3$ just by taking countable unions, intersections and complements, one says that the above sets form a generating system *of $\mathcal{F}_i, i\leq 3$.*

Presence-Exercise 3.II:(Stopping times)

Consider $X=\big(X_n\big)_{n\in\mathbb{N}}$ a Markov Chain with countable state space I on some probability space $\big(\Omega,\mathcal{F},\mathbb{P}\big)$ where $\mathcal{F}=\bigcup_{i\in\mathbb{N}}\mathcal{F}_i$ is the natural filtration of the Markov Chain X. Let T be a stopping time.

1. Is $T+3$ again a stopping time, what about $T-3$ for $T>3$?
2. Is the product of two stopping times $(S\cdot T):\Omega\to\mathbb{N}\cup\{0\}$ if $S,T\geq 0$, again a stopping time?
3. Does b) hold if $S,T>1$?

Presence-Exercise 3.III:

Consider $X=\big(X_n\big)_{n\in\mathbb{N}}$ a Markov Chain with countable state space I on some probability space $\big(\Omega,\mathcal{F},\mathbb{P}\big)$ where $\mathcal{F}=\bigcup_{i\in\mathbb{N}}\mathcal{F}_i$ is the natural filtration of the Markov Chain X.

1. Show that a filtration is always increasing
2. Show:
   A mapping $T:\Omega\to\mathbb{N}$ is a stopping time $\iff$ for all $n\in\mathbb{N}:\ \{T\leq n\}\in\mathcal{F}_n$.