Dr. Kilian Raschel
Daniel Altemeier
Department of Mathematics
Bielefeld University

SS 2011

Stochastik 2 - Presence-Exercises 5:

Presence-Exercise 5.I:

Consider a randomly moving particle on the corners of a triangle generated by the points A, B, C (Think of $A = (0,0), B = (0,1)$ and $C = (1,0)$). Per unit of time our particle jumps from one of the points to one of the neighbouring points.

(a) Show that in the symmetric case (particle runs as likely clockwise as counterclockwise), there is an invariant distribution and the Markov Chain is reversible.

(b) Which results remain in the asymmetrical case? Here, do only consider the case, where the particle prefers moving clockwise with 'intensity' 3/4 to 1/4.

Presence-Exercise 5.II:

Each morning a student takes one of the three books he owns from his shelf. The probability that he chooses book i as α_i where $0 < \alpha_i < 1$ for $i = 1, 2, 3$ and choices on successive days are independent. In the evening he replaces the book at the left-hand end of the shelf. If p_n denotes the probability that on day n the student finds the books in the order 1,2,3, from left to right, show that, irrespective of the initial arrangement of the books, p_n converges as $n \to \infty$, and determine the limit.

Presence-Exercise 5.III:

Remember Presence-Exercise 4.I:

Gas molecules move about randomly in a box which is divided into two halves symmetrically by a partition. A hole is made in the partition. Suppose there are N molecules in the box of which exactly one passes the partition each time-period. Show that the number of molecules on one side of the partition just after a molecule has passed through the hole evolves as a Markov Chain. What are the transition probabilities? What is the invariant distribution of this chain? Explain in detail the construction of your Markov Chain.

For $N = 4$ compute the invariant distribution and show that it is reversible.