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Stochastik 2 - Presence-Exercises 6:

Presence-Exercise 6.I:

Show *Proposition B.1.8*:
A CTMC $X = \big(X_t\big)_{t\in[0,\infty)}$ *is a* Poisson process *if and only if* X *satisfies the* (PP_λ)*-property.*

Presence-Exercise 6.II

Let X be Poisson-distributed on some probability space $\big(\Omega, \mathcal{F}, \mathbb{P}\big)$ and let Y denote the number of successes in X Bernoulli-experiments with parameter of succes $p \in (0, 1)$. Show that Y and $X - Y$ are independently Poisson-distributed with parameter λp and $\lambda(1 - p)$.

Presence-Exercise 6.III:

Remember Presence-Exercise 4.I: *Gas molecules move about randomly in a box which is divided into two halves symmetrically by a partition. A hole is made in the partition. Suppose there are* N *molecules in the box of which exactly one passes the partition each time-period. Show that the number of molecules on one side of the partition just after a molecule has passed through the hole evolves as a Markov Chain. What are the transition probabilities? What is the invariant distribution of this chain? Explain in detail the construction of your Markov Chain.*

Now, forget about the restriction of one particle per time-unit but consider i.i.d. exponentially distributed waiting times between jumps. Write down the Q-matrix of this problem.