

Stochastik 2 - Presence-Exercises 7:

Presence-Exercise 7.I:

Customer arrive at a store at the instants of a Poisson process of rate 2. At the door, two representatives separately demonstrate the same product to anybody entering the store. Each demonstration takes a time which is exponentially distributed with parameter 1, and is independent of other demonstrations. After the demonstration the customer enters the store. If both representatives are busy the customer goes directly into the store.

If both representatives are free at $t = 0$, show that the probability that both are busy at $t > 0$ is

$$\frac{2}{5} - \frac{2}{3} \exp(-2t) + \frac{4}{15} \exp(-5t).$$

You don't want to count customers *in* the shop.

Presence-Exercise 7.II:

Let $(N(t))_{t \geq 0}$ be a Poisson-process with parameter λ , define

$$Z(t) := (-1)^{N(t)}, t \geq 0.$$

Show that

$$\mathbb{P}[Z(s) = Z(t)] = \frac{1}{2}(1 + \exp(-2\lambda(t - s))), \quad \forall s \geq 0 \text{ and } t > s.$$