Dr. Kilian Raschel
Daniel Altemeier
Department of Mathematics
Bielefeld University

SS 2011

Stochastik 2 - Presence-Exercises 8:

Presence-Exercise 8.I:

Let $\left(X_t\right)_{t\geq 0}$ be a CTMC with Q-matrix

$$Q = \begin{pmatrix} -\mu & \lambda_1 & \dots & \lambda_n \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 0 \end{pmatrix}.$$

1. Compute the tansition-matrix (more precisely the whole transition semigroup) $P_{ij}(t), t \geq 0$.
2. Draw the transition graph at time t as well as the *rate transition graph*

Exercise 8.II

Consider the CTMC with Q-matrix

$$Q = \begin{pmatrix} -\alpha & \alpha \\ \beta & -\beta \end{pmatrix}, \quad \alpha, \beta > 0$$

1. Write down the backward and forward equations. Solve either the backward or the forward equation for the transition probabilities $p_{ij}(t), j = 1, 2$. Solve one of them and check that your solutions also solves for the other one.
   Hint: One equation is easier to solve than the other, note furthermore that the solution of $y'(t) = ay(t) + b, t \geq 0$ is given by $y(t) = C \exp(at) - b/a, C \in \mathbb{R}$.
2. Calculate Q^n by setting up recurrence relations for its entries and hence find
   $$\exp(tQ) = \sum_{n=0}^{\infty} \frac{t^n Q^n}{n!},$$
   compare your answer with that to part (a).
3. Solve the equation $\xi Q = 0$ for ξ and verify that $p_{i,j}(t) \to \xi_j$ as $t \to \infty$. What is the interpretation of this?