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Stochastik 2 - Presence-Exercises 9:

Presence-Exercise 9.I:

Let X be some CTMC with Q-matrix

$$Q = \begin{pmatrix} -2 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 1 & -3 \end{pmatrix}.$$

Compute the probability $p_{11}(t)$ to return to or stay in state 1 in time t 1. Use the instructions below:

step 1: Compute the eigen values of the matrix

step 2: The matrix Q can be diagonalized, i.e. there is an invertible U such that:

$$Q = U \begin{pmatrix} y_1 & 0 & 0 \\ 0 & y_2 & 0 \\ 0 & 0 & y_3 \end{pmatrix} U^{-1},$$

where y_1, y_2, y_3 are the eigenvalues of Q. Deduce that $\exp(Q)$ is diagonalizable with the same matrix U.

step 3: Deduce an explicit functional term for $p_{11}(t)$ depending on the entries of U and terms of the form $\alpha_i \exp(\beta_i t), \alpha_i, \beta_i \in \mathbb{R}, i = 1, 2, 3$.

step 4: Now, we know $P(0)$, $P'(0)$ and by the BE we know what $P''(0)$ 'looks like'.

step 5: Denote a system of linear equations and solve it for the parameters α_1, α_2 and α_3.

Presence-Exercise 9.II:

For the Poisson-process with parameter λ we want to compute the transition semi-group.

1. Denote the BE and the FE,
2. Solve one of them and show that the solution also satisfies the other.
3. Is the solution unique?