# Das BDF-Verfahren:

Anfangswertaufgabe:

$$u'(t) = f(t, u(t)),\ t \in [t_0, t_E],\ u(t_0) = u_0 \in \mathbb{R}^n, \quad u: \mathbb{R} \to \mathbb{R}^n$$

BDF-Verfahren (der Ordnung $m$): $h > 0$, $t_i = t_0 + h \cdot i$, $i = 0, \dots, M$, $t_M \leq t_E$

$$\frac{1}{h} \sum_{\nu=0}^{m} \alpha_\nu u^{j+\nu} = f(t_{j+m}, u^{j+m}), \quad j = 0, \dots, M-m$$

Gegeben: $\alpha_0, \dots, \alpha_m$ Koeffizienten

$h$ Schrittweite

$u^0, \dots, u^{m-1}$ Startdaten (z.B. durch ESV bestimmen), $u^j \approx u(t_j)$

Koeffizienten:

| | $m$ | $\alpha_0$ | $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | $\alpha_4$ | $\alpha_5$ | |
|---|---|---|---|---|---|---|---|---|
| BDF(1) = Euler-Cauchy | 1 | 1 | $-1$ | | | | | A-stabil |
| BDF(2) | 2 | $\frac{3}{2}$ | $-2$ | $\frac{1}{2}$ | | | | |
| BDF(3) | 3 | $\frac{11}{6}$ | $-3$ | $\frac{3}{2}$ | $-\frac{1}{3}$ | | | A($\alpha$)-stabil |
| BDF(4) | 4 | $\frac{25}{12}$ | $-4$ | 3 | $-\frac{4}{3}$ | $\frac{1}{4}$ | | |
| BDF(5) | 5 | $\frac{137}{60}$ | $-5$ | 5 | $-\frac{10}{3}$ | $\frac{5}{4}$ | $-\frac{12}{60}$ | |
| | | | | | | | | instabil |

$\uparrow$ Konsistenzordnung

$$\frac{1}{h} \cdot \left(u^j - u^{j+1}\right) = f(t_{j+1}, u^{j+1}) \qquad , j = 0, \dots, M-1$$

$$\frac{1}{h} \cdot \left(\frac{3}{2} u^j - 2u^{j+1} + \frac{1}{2} u^{j+2}\right) = f(t_{j+2}, u^{j+2}) \qquad , j = 0, \dots, M-2$$

$$\frac{1}{h} \cdot \left(\frac{11}{6} \cdot u^j - 3u^{j+1} + \frac{3}{2} u^{j+2} - \frac{1}{3} u^{j+3}\right) = f(t_{j+3}, u^{j+3}) \qquad , j = 0, \dots, M-3$$

$$\frac{1}{h} \cdot \left(\frac{25}{12} u^j - 4u^{j+1} + 3u^{j+2} - \frac{4}{3} u^{j+3} + \frac{1}{4} u^{j+4}\right) = f(t_{j+4}, u^{j+4}) \qquad , j = 0, \dots, M-4$$

$$\frac{1}{h} \cdot \left(\frac{137}{60} u^j - 5u^{j+1} + 5u^{j+2} - \frac{10}{3} u^{j+3} + \frac{5}{4} u^{j+4} - \frac{1}{5} u^{j+5}\right) = f(t_{j+5}, u^{j+5}) \qquad , j = 0, \dots, M-5$$