Heat Transfer in a Coffee Cup with Comsol Multiphysics

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1. Introduction and Mathematical Setting

2. Model Wizard

Start Comsol Multiphysics.
To start Comsol Multiphysics 5.0 open the **Terminal** and enter

- **comsol -ckl**

Model Wizard.

- In the **New** window, click **Model Wizard**.
- In the **Model Wizard** window, click **3D** in the **Select Space Dimension** menu.
- In the **Select Physics** tree, select **Mathematics>PDE Interfaces>Coefficient Form PDE (c)**.
- Click **Add**.
- Next, locate the **Dependent Variables** section.
- In the **Field name** text field, type **T**.
- In the **Dependent Variables** text field, type also **T**.
- Next, locate the **Units** section.
- From the **Dependent variable quantity** list, choose **Temperature (K)**.
- From the **Source term quantity** list, choose **Heat source (W/m^3)**.
- Click **Study**.
- In the **Select Study** tree, select **Preset Studies>Time Dependent**.
- Click **Done**.

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Unit System.

- In the **Model Builder** window, click **Untitled.mph** ***(root)***.
- In the **Settings** window for Root, locate the **Unit System** section.
- From the **Unit System** list, choose **SI**.

Some Advanced Settings.
Hint: In the **Model Builder** window you should click on the **Show** icon and enable everything that is possible from the menu: **Expand Sections** (**Equation View**, **Override and Contribution**, **Discretization**, **Stabilization**, **Advanced Physics Options**, **Advanced Study Options** and **Advanced Results Options**). Done this, click **Expand All** icon.

3. Parameters and Variables

Parameters.

- In the **Model Builder** window, expand the **Global** node, right-click **Definitions** and select **Parameters**. (Alternatively: On the **Model** toolbar, click **Parameters**.)
- In the **Settings** window for Parameters, locate the **Parameters** section.
- In the table, enter the following settings:

Name	Expression	Value	Description
L1	0.073[m]	0.073 m	innerer Durchmesser der Tasse
L2	0.079[m]	0.079 m	äußerer Durchmesser der Tasse
L3	0.055[m]	0.055 m	Größe des Haltegriffs (bzw. Durchmesser des Toruses
L4	0.007[m]	0.007 m	Dicke (bzw. innerer Durchmesser) des Haltegriffs
H1	0.08[m]	0.08 m	max. Füllhöhe der Tasse
H2	0.008[m]	0.008 m	Höhe des Bodens der Tasse
H3	0.07[m]	0.07 m	tatsächliche Füllhöhe
H4	0.046[m]	0.046 m	Abstand mittlerer Teil des Haltegriffes vom Tassenboden

Variables.

- In the **Model Builder** window, expand the **Component 1 (comp1)** node, right-click **Definition** and select **Variables**.
- In the **Settings** window for Variables, locate the **Variables** section.
- In the table, enter the following settings:

Name	Expression	Unit	Description
T0_wasser	(273.15+60)[K]	K	Temp. der Flüssigkeit zu Meßbeginn
T0_tasse	(273.15+23)[K]	K	Temp. der Tasse zu Meßbeginn
T_luft	(273.15+23)[K]	K	Temp. der Raumluft zu Meßbeginn
kappa_luft	10[W/(m^2·K)]	W/(m^2·K)	Wärmeübergangskoeffizient Luft
rho_wasser	995[kg/m^3]	kg/m^3	Dichte Luft (bei 30 °C)
c_wasser	4182[J/(kg·K)]	J/(kg·K)	spez. Wärmekapazität Wasser (20 °C)
lambda_wasser	0.597[W/(m·K)]	W/(m·K)	Wärmeleitfähigkeit Wasser (20 °C)
rho_glas	2500[kg/m^3]	kg/m^3	Dichte Glas
c_glas	600[J/(kg·K)]	J/(kg·K)	spez. Wärmekapazität Glas
lambda_glas	0.76[W/(m·K)]	W/(m·K)	Wärmeleitfähigkeit Glas
rho_porzellan	2300[kg/m^3]	kg/m^3	Dichte Porzellan
c_porzellan	730[J/(kg·K)]	J/(kg·K)	spez. Wärmeleitfähigkeit Porzellan
lambda_porzellan	1.03[W/(m·K)]	W/(m·K)	Wärmeleitfähigkeit Porzellan
g_wasser	0[m^2/s^3]	m^2/s^3	
g_glas	0[m^2/s^3]	m^2/s^3	
g_porzellan	0[m^2/s^3]	m^2/s^3	

4. Geometry

Work Plane 1 (middle part of the coffee cup).

- In the **Model Builder** window, expand the **Component 1 *(comp1)*** node, right-click **Geometry** and select **Work Plane**.
- In the **Settings** window for Work Plane, locate the **Plane Definition** section.
- In the **z-coordinate** text field, type **H2**.
- In the **Model Builder** window, expand the **Component 1 *(comp1)*>Geometry 1>Work Plane 1 *(wp1)*** node, right-click **Plane Geometry** and select **Circle**.
- In the **Settings** window for Circle 1, locate the **Size and Shape** section.
- In the **Radius** text field, type **L2/2**.
- Right-click **Plane Geometry** again and select **Circle**.
- In the **Settings** window for Circle 2, locate the **Size and Shape** section.
- In the **Radius** text field, type **L1/2**.
- Right-click **Plane Geometry** once more and select **Booleans and Partions>Difference**.
- In the **Settings** window for Difference, locate the **Difference** section.
- In the subsection **Objects to add**, the active button should be activated (green). Now, switching to the **Graphics** window and left-click on the larger circle. In the **Objects to add** list, the entry **c1** now appears.
- Next, activate the activ button in the subsection **Objects to substract**. Switching once more to the **Graphics** window and left-click on the smaller (inner) circle. In the **Objects to substract** list, the entry **c2** now appears.
- In the **Model Builder** window, right-click on **Work Plane 1 *(wp1)*** and select **Extrude**.
- In the **Settings** window for Extrude, locate the **Distance from Plane** section.
- In the **Distances (m)** table substitute 1 by **H3**.
- In the **Model Builder** window, right-click on **Extrude 1 *(ext1)*** and select **Build Selected** to check the result.

Work Plane 2 (top of the coffee cup).

- In the **Model Builder** window, expand the **Component 1 *(comp1)*** node, right-click **Geometry** and select **Work Plane**.
- In the **Settings** window for Work Plane, locate the **Plane Definition** section.
- In the **z-coordinate** text field, type **H2+H3**.
- In the **Model Builder** window, expand the **Component 1 *(comp1)*>Geometry 1>Work Plane 2 *(wp2)*** node, right-click **Plane Geometry** and select **Circle**.
- In the **Settings** window for Circle 1, locate the **Size and Shape** section.
- In the **Radius** text field, type **L2/2**.
- Right-click **Plane Geometry** again and select **Circle**.
- In the **Settings** window for Circle 2, locate the **Size and Shape** section.
- In the **Radius** text field, type **L1/2**.
- Right-click **Plane Geometry** once more and select **Booleans and Partions>Difference**.
- In the **Settings** window for Difference, locate the **Difference** section.
- In the subsection **Objects to add**, the active button should be activated (green). Now, switching to the **Graphics** window and left-click on the larger circle. In the **Objects to add** list, the entry **c1** now appears.
- Next, activate the activ button in the subsection **Objects to substract**. Switching once more to the **Graphics** window and left-click on the smaller (inner) circle. In the **Objects to substract** list, the entry **c2** now appears.
- In the **Model Builder** window, right-click on **Work Plane 2 *(wp2)*** and select **Extrude**.
- In the **Settings** window for Extrude, locate the **Distance from Plane** section.
- In the **Distances (m)** table substitute 1 by **H1-H3**.
- In the **Model Builder** window, right-click on **Extrude 2 *(ext2)*** and select **Build Selected** to check the result.

Cylinder (bottom of the coffee cup).

- In the **Model Builder** window, right-click on **Geometry 1** and select **Cylinder**.
- In the **Settings** window for Cylinder, locate the **Size and Shape** section.
- In the **Radius** text field, type **L2/2** and in the **Height** text field **H2**.
- In the **Model Builder** window, right-click on **Cylinder 1 *(cyl1)*** and select **Build Selected** to check the result.

Torus (handle of the cup).

- In the **Model Builder** window, right-click on **Geometry 1** and select **More Primitives>Torus**.
- In the **Settings** window for Cylinder, locate the **Size and Shape** section.
- In the **Major radius** text field, type **L3/2**, in the **Minor radius** text field **L4/2** and in the **Revolution angle** text field **180**.
- Next, locate the **Position** section.
- In the **x** text field, type **-L2/2+0.001**, in the **y** text field **0** and in the **z** text field **H4**.
- Finally, locate the **Axis** section.
- As **Axis type** choose **y-axis**.

- In the **Model Builder** window, right-click on **Torus 1** ***(tor1)*** and select **Build Selected** to check the result.

Union.

- In the **Model Builder** window, right-click on **Geometry 1** and select **Booleans and Partions>Union**.
- In the **Settings** window for Union, locate the **Union** section.
- In the subsection **Input objects**, the active button should be activated (green). Now, switching to the **Graphics** window and left-click on the top, the middle, the buttom and on the handle of the cup. In the **Input objects** list, the entries **ext1**, **ext2**, **cyl1** and **tor1** now appears.

Cylinder (liquid in the cup).

- In the **Model Builder** window, right-click on **Geometry 1** and select **Cylinder**.
- In the **Settings** window for Cylinder 2, locate the **Size and Shape** section.
- In the **Radius** text field, type **L1/2** and in the **Height** text field **H3**.
- Next, locate the **Position** section.
- In the **x** text field, type **0**, in the **y** text field **0** and in the **z** text field **H2**.
- In the **Model Builder** window, right-click on **Form Union** ***(fin)*** and select **Build Selected** to check the overall result.
- The complete geometry is shown in Figure 4.1.

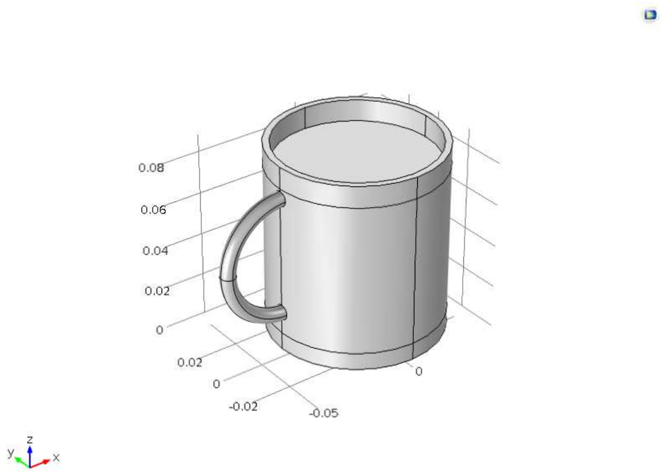

ABBILDUNG 4.1. Geometry (spatial domain) of the coffee cup that is filled with a fluid (coffee)

5. The Heat Equation

Partial differential equation.

- In the **Model Builder** window, expand the **Component 1 (comp 1)>Coefficient Form PDE** ***(c)*** node, then click **Coefficient Form PDE 1**.
- We are now in **Settings** window for Coefficient Form PDE.

- In the **Diffusion Coefficient** c text field, type **lambda_glas**.
- In the **Source Term** f text field, type **rho_glas*g_glas**.
- In the **Damping or Mass Coefficient** d_a text field, type **rho_glas*c_glas**.
- In the **Model Builder** window, right-click **Coefficient For PDE *(c)*** and select **Coefficient Form PDE**.
- In the **Model Builder** window, click **Coefficient Form PDE 2**.
- In the **Settings** window for Coefficient Form PDE 2, locate the **Domain Selection** section.
- As **Selection** choose **Manual**. The domain list should contain only domain **7**, that corresponds to the liquid.
- In the **Diffusion Coefficient** c text field, type **lambda_wasser**.
- In the **Source Term** f text field, type **rho_wasser*g_wasser**.
- In the **Damping or Mass Coefficient** d_a text field, type **rho_wasser*c_wasser**.

Boundary conditions.

- In the **Model Builder** window, right-click **Coefficient For PDE *(c)*** and select **Flux/Source**.
- In the **Model Builder** window, click **Flux/Source 1**.
- In the **Settings** window for Flux/Source 1, locate the **Boundary Selection** section.
- As **Selection** choose **Manual**. The boundary list should contain the boundaries **1-13,27,28,30,36-41,43,45-48**. These correspond to all faces that have contact with the air.
- In the **Boundary Flux/Source** g text field, type **-kappa_luft*(T-T_luft)**.

Initial values.

- In the **Model Builder** window, select **Coefficient For PDE *(c)*>Initial Values 1**.
- In the **Settings** window for Initial Values 1, locate the **Initial Values** section.
- For **Initial value for T** specify **T0_tasse**.
- In the **Model Builder** window, right-click **Coefficient For PDE *(c)*** and select **Initial Values**.
- In the **Settings** window for Initial Values 2, locate the **Domain Selection** section.
- As **Selection** choose **Manual**. The domain list should contain only domain **7**, that corresponds to the liquid.
- In the **Settings** window for Initial Values 2, locate the **Initial Values** section.
- For **Initial value for T** specify **T0_wasser**.
- The initial temperature distribution $T(x,0) = T_0(x)$ at initial time $t = 0$ is shown in Figure 5.1.

Spatial discretization.

- In the **Model Builder** window, click **Coefficient Form PDE *(c)***.
- In the **Settings** window for Coefficient Form PDE, locate the **Discretization** section.
- As **Shape function type** select **Lagrange** and as **Element order** choose **Linear**.

6. Mesh, Study Settings and Computation

Generating Mesh.

- In the **Model Builder** window, click **Mesh 1**.

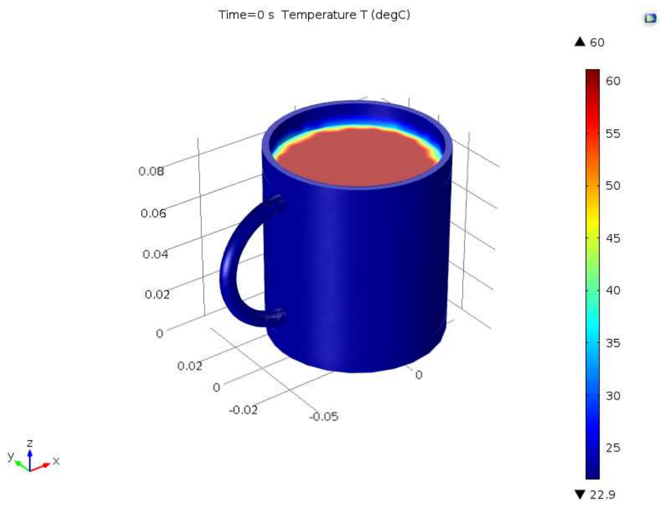

ABBILDUNG 5.1. Initial temperature distribution T (in °C) of the coffee cup at initial time $t = 0$ (in s)

- In the **Settings** window for Mesh, locate the **Mesh Settings** section.
- As **Sequence type** select **User-controlled mesh**.
- In the **Model Builder** window, expand the **Mesh 1** node, then click **Size**.
- In the **Settings** window for Size, locate the **Element Size Parameters** section.
- In the **Maximum element size** text field, type **0.011**.

Study Settings.

- In the **Model Builder** window, expand the **Study 1** node, then click **Step 1: Time Dependent**.
- In the **Settings** window for Time Dependent, locate the **Study Settings** section.
- In the **Times** text field, type **range(0,0.1,60)**.
- **Enable** the checkbox for **Relative Tolerance**.
- In the **Model Builder** window, right-click **Solver Configurations** and select **Show Default Solver**.
- In the **Model Builder** window, expand the **Study 1>Solver Configurations>Time-Dependent Solver 1** node and click **Time-Dependent Solver 1**.
- In the **Settings** for Time-Dependent Solver 1, locate the **Absolute Tolerance** section.
- As **Global method** choose **unscaled**.
- In the **Settings** for Time-Dependent Solver 1, now locate the **Time Stepping** section.
- As **Method** choose **BDF**, as **Steps taken by solver** choose **Intermediate** and as **Maximum BDF oder** choose **2**.
- In the **Model Builder** window, expand the **Study 1>Solver Configurations>Time-Dependent Solver 1** node and click **Fully Coupled 1**.
- In the **Setting** for Fully Coupled 1, locate the **Method and Termination** section.
- As **Nonlinear method** choose **Automatic (Newton)**.

Computation.

- In the **Model Builder** window, expand the **Study 1** node, right-click on **Step 1: Time-Dependent** and select **Compute Selected Step**.

7. Postprocessing and Graphical Output

Surface Plot.

- In the **Model Builder** window, expand the **Results** node, then right-click **Results>3D Plot Group 1** and select **Surface** from the menu.
- Next, right-click on **Results>3D Plot Group 1>Slice 1** and select **Delete** from the menu.
- Now, click on **Results>3D Plot Group 1**.
- In the **Settings** window for 3D Plot Group 1, first locate the **Data** section. For **Data set** select **Study 1/Solution 1** and for **Time (s)** select **60**.
- Next, locate the **Plot Settings** section. **Disable** the check box for **Plot data set edges**.
- In the **Model Builder** window, click on **Results>3D Plot Group 1>Surface 1**.
- In the **Settings** window for Surface 1, first locate the **Data** section and select **From Parent** for **Data set**.
- Next, locate the **Expression** section. For **Expression** type **T**, for **Unit** select **degC** from the list and **disable** the check box for **Description**.
- Now, locate the **Title** section. For **Title type** select **custom**. In the **Type and data** subsection **disable** the check boxes for **Type**, **Description** and **Expression**, and **enable** the check box for **Unit**. In the text field for **Prefix** type **Temperatur T**.
- Finally, locate the **Range** section. **Enable** the check boxes for **Manual color range** and **Manual data range**. In the text fields for **Minimum** type **22** and for **Maximum** type **61**.
- The result is shown in Figure 7.1.

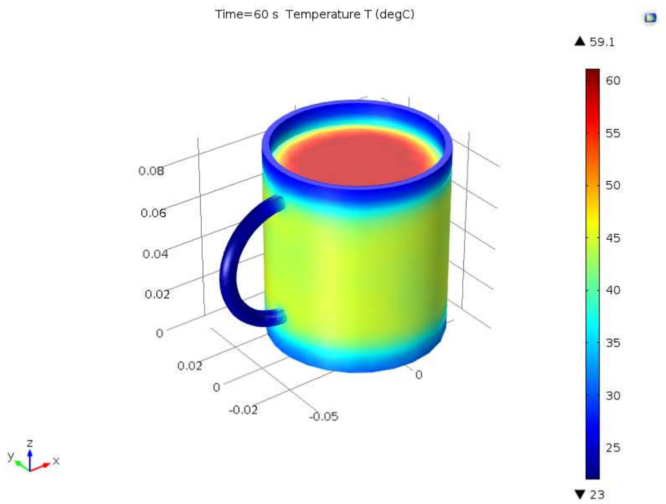

ABBILDUNG 7.1. Surface of the heat distribution (temperature) T (in °C) of the coffee cup at time $t = 60$ (in s)

Slice Plot.

- In the **Model Builder** window, right-click on **Results** and select **3D Plot Group** from the menu.
- Next, right-click on **Results>3D Plot Group 2** and select **Slice** from the menu.
- Now, click **Results>3D Plot Group 2**.
- In the **Settings** window for 3D Plot Group 2, locate the **Data** section. As **Data set** select **Study 1/Solution 1** and as **Time (s)** select **60** from the list.
- In the **Model Builder** window, click **Results>3D Plot Group 2>Slice 1**.
- In the **Settings** window for Slice 1, first locate the **Data** section and select **From Parent** for **Data set**.
- Next, locate the **Expression** section. For **Expression** type **T**, for **Unit** select **degC** from the list and **disable** the check box for **Desciption**.
- Now, locate the **Title** section. For **Title type** select **custom**. In the **Type and data** subsection **disable** the check boxes for **Type**, **Description** and **Expression**, and **enable** the check box for **Unit**. In the text field for **Prefix** type **Temperatur T**.
- Next, locate the **Plane Data** section, select **xy-plane** for **Plane** and type **4** in the **Planes** text field.
- Finally, locate the **Range** section. **Enable** the check boxes for **Manual color range** and **Manual data range**. In the text fields for **Minimum** type **22** and for **Maximum** type **61**.
- The result is shown in Figure 7.2.

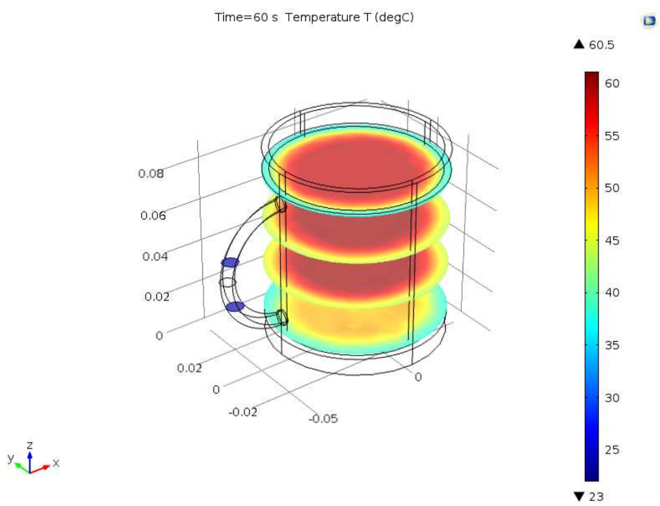

ABBILDUNG 7.2. Slices of the heat distribution (temperature) T (in °C) of the coffee cup at time $t = 60$ (in s)

Animation.

- In the **Model Builder** window, right-click on **Results>Export** and select **Animation** from the menu.
- Click on **Results>Export>Animation 1**.

- In the **Settings** window for Animation 1, first locate the **Scene** section and select **3D Plot Group 1** for **Subject**.
- Next, locate the **Output** section. As **Output type** select **Movie**, as **Format** select **GIF** and in the **File name** text field type **coffee.gif**. In the **Frames per second** text field type **10**.
- Now, locate the **Frames** section. As **Frame selection** select **Number of frames** and in the **Number of frames** text field type **100**. In the **Width** text field type **800** and in the **Height** text field **600**.
- Finally, locate the **Layout** section and **enable** all check boxes, i.e. for **Include**, **Title**, **Color legend**, **Grid**, **Axis orientation** and **Logotype**. In the **Font size** text field type **10**.
- Now, click on the **Export (F8)** symbol to create the movie. The symbol can be found in the header of the **Settings** window.

8. Save the Model

Save File.

- Select **File>Save As...**.
- Select a desired folder, where the model should be saved, and enter **CoffeeCup.mph** as the **Name** for the model.
- Click **OK**.