Ordinary Differential Equations with Comsol Multiphysics

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1. Introduction and Mathematical Setting

In the following we explain how to solve the following ordinary differential equation

$$u''(t) = -\frac{gR^2}{\left(u(t)+R\right)^2},\ t > t_0,$$
$$u(t_0) = u_0, \quad u'(t_0) = v_0,$$

where the gravitational acceleration g (in $\frac{\mathrm{m}}{\mathrm{s}^2}$), the Earth's radius R (in m), the initial time t_0 (in s), the initial height u_0 (in m) and the initial velocity v_0 (in $\frac{\mathrm{m}}{\mathrm{s}}$) are given. We seak for a time-dependent function u (in m) that descibes the height of the body at time t (in s) measured from the Earth's surface.
For our simulation we use the following values

$$g = 10\,\frac{\mathrm{m}}{\mathrm{s}^2}, \quad R = 10^7\,\mathrm{m}, \quad t_0 = 0\,\mathrm{s}, \quad u_0 = 0\,\mathrm{m}, \quad v_0 = 10\,\frac{\mathrm{m}}{\mathrm{s}}.$$

2. Model Wizard

Start Comsol Multiphysics.
To start Comsol Multiphysics 5.0 open the **Terminal** and enter

- **comsol -ckl**

Model Wizard.

- In the **New** window, click **Model Wizard**.
- In the **Model Wizard** window, click **0D** in the **Select Space Dimension** menu.
- In the **Select Physics** tree, select **Mathematics>ODE and DAE interfaces>Global ODEs and DAEs (ge)**.
- Click **Add**.

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- Click **Study**.
- In the **Select Study** tree, select **Preset Studies>Time Dependent**.
- Click **Done**.

Unit System.

- In the **Model Builder** window, click **Untitled.mph *(root)***.
- In the **Settings** window for Root, locate the **Unit System** section.
- From the **Unit System** list, choose **SI**.

Some Advanced Settings.
Hint: In the **Model Builder** window you should click on the **Show** icon and enable everything that is possible from the menu: **Expand Sections** (**Equation View**, **Override and Contribution**, **Discretization**, **Stabilization**, **Advanced Physics Options**, **Advanced Study Options** and **Advanced Results Options**). Done this, click **Expand All** icon.

3. Ordinary Differential Equation

- In the **Model Builder** window, expand the **Component 1>Global ODEs and DAEs (ge)** node, then click **Global Equations 1**.
- In the **Settings** window for Global Equations, locate the **Global Equations** section.
- In the table, enter the following settings:

Name	$f(u, ut, utt, t)$ (1)	Initial value (u_0) (1)	Initial value (u_t0) (1)	Description
u	$utt - F$	$u0$	$v0$	

- In the **Settings** window for Global Equations, locate the **Units** section.
- For **Dependent variable quantity** specify **Length (m)**.
- For **Source term quantity** specify **Acceleration (m/s^2)**

4. Parameters and Variables

Parameters.

- In the **Model Builder** window, expand the **Global** node, right-click **Definitions** and select **Parameters**. (Alternatively: On the **Model** toolbar, click **Parameters**.)
- In the **Settings** window for Parameters, locate the **Parameters** section.
- In the table, enter the following settings:

Name	Expression	Value	Description
g	10	10	gravitational acceleration (in m/s^2)
R	10^7	$1.0000E7$	Earth's radius (in m)
$t0$	0	0	initial time (in s)
$u0$	0	0	initial height (in m)
$v0$	10	10	initial velocity (in m/s)

Variables.

- In the **Model Builder** window, expand the **Component 1 (comp1)** node, then click **Global Equations 1**. (Alternatively: On the **Model** toolbar, click **Variables** the **Local Variables**.)
- In the **Settings** window for Variables, locate the **Variables** section.
- In the table, enter the following settings:

Name	Expression	Unit	Description
F	$-g*R\hat{}2/(u+R)\hat{}2$	$1/m^2$	right hand side of 2nd order ODE

Hint: Note that the **variables** must be chosen **local**, not global.

5. Study Settings and Computation

Study Settings.

- In the **Model Builder** window, expand the **Study 1** node, then click **Step 1: Time Dependent**.
- In the **Settings** window for Time Dependent, locate the **Study Settings** section.
- In the **Times** text field, type **range(t0,0.1,2)**.

Computation.

- On the **Model** toolbar, click **Compute**.

6. Postprocessing and Graphical Output

The time-height plot.

- In the **Model Builder** window, expand the **Results** node, then click **1D Plot Group 1**.
- In the **Settings** window for 1D Plot Group, locate the **Title** section.
- From the **Title type** list, choose **Manual**. In the **Title** text field, type **Vertical throw and free fall of a body**.
- Locate the **Plot Settings** section. **Mark** both check boxes, **x-axis label** and **y-axis label**.
- In the **x-axis label** text field, enter **Time t (s)**. In the **y-axis label** text field, enter **Height u (m)**.
- In the **Model Builder** window, expand the **Results>1D Plot Group 1** node, then click **Global 1**.
- In the **Settings** window for Global, locate the **Legends** section.
- **Clear** the checkbox **Show legends**.
- The result is shown in Figure 7.1.

The time-velocity plot.

- In the **Model Builder** window, right-click **Results** and select **1D Plot Group**.
- In the **Model Builder** window, expand the **Results** node, then click **1D Plot Group 2**.
- From the **Title type** list, choose **Manual**. In the **Title** text field, type **Vertical throw and free fall of a body**.
- Locate the **Plot Settings** section. **Mark** both check boxes, **x-axis label** and **y-axis label**.
- In the **x-axis label** text field, enter **Time t (s)**. In the **y-axis label** text field, enter **Velocity u_t (m)**.

- In the **Model Builder** window, expand **Results**, right-click **1D Plot Group 2** and select **Global**.
- In the **Settings** window for Global, locate the **y-Axis Data** section.
- In the table, enter the following settings:

Expression	Unit	Description
comp1.ut	m/s	State Variable u, first time derivative

- In the **Settings** window for Global, locate the **Legends** section.
- **Enable** the checkbox **Show legends**.
- The result is shown in Figure 7.2.

The time-acceleration plot.

- In the **Model Builder** window, right-click **Results** and select **1D Plot Group**.
- In the **Model Builder** window, expand the **Results** node, then click **1D Plot Group 3**.
- From the **Title type** list, choose **Manual**. In the **Title** text field, type **Vertical throw and free fall of a body**.
- Locate the **Plot Settings** section. **Mark** both check boxes, **x-axis label** and **y-axis label**.
- In the **x-axis label** text field, enter **Time t (s)**. In the **y-axis label** text field, enter **Acceleration u_tt (m)**.
- In the **Model Builder** window, expand **Results**, right-click **1D Plot Group 3** and select **Global**.
- In the **Settings** window for Global, locate the **y-Axis Data** section.
- In the table, enter the following settings:

Expression	Unit	Description
comp1.utt	m/s^2	State Variable u, second time derivative

- In the **Settings** window for Global, locate the **Legends** section.
- **Enable** the checkbox **Show legends**.
- The result is shown in Figure 7.3.

7. Save the Model

Save File.

- Select **File>Save As...**.
- Select a desired folder, where the model should be saved, and enter **ODE.mph** as the **Name** for the model.
- Click **OK**.

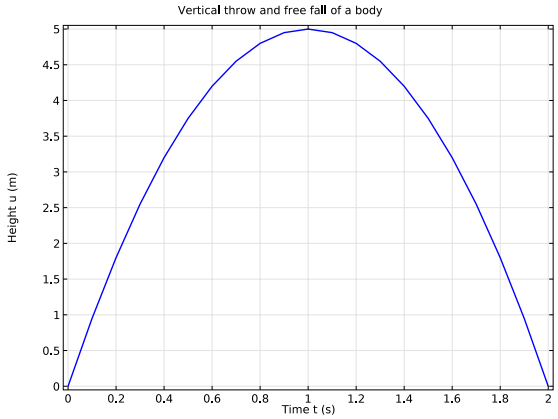

FIGURE 7.1. Height $u(t)$ (in m) of the body at time t (in s)

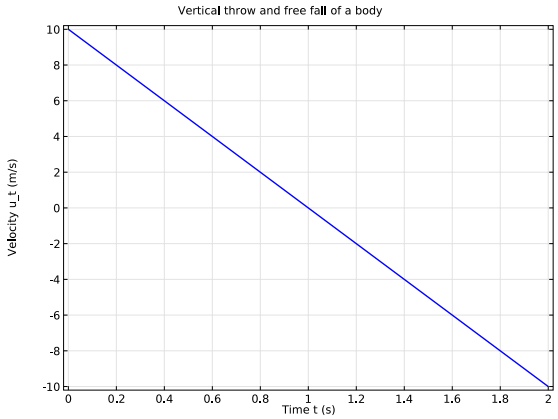

FIGURE 7.2. Velocity $v(t) = u'(t)$ (in $\frac{\mathrm{m}}{\mathrm{s}}$) of the body at time t (in s)

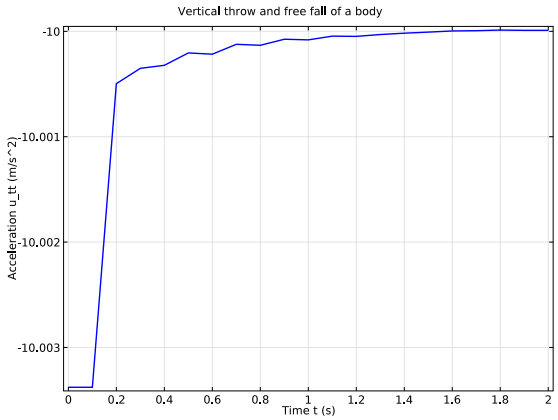

FIGURE 7.3. Acceleration $a(t) = u''(t)$ (in $\frac{\mathrm{m}}{\mathrm{s}^2}$) of the body at time t (in s)