Groups of local characteristic p

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Definition

Let G be a group and p a prime.

- A *p-local subgroup* of G is the normalizer of a non-trivial p-subgroup of G.
- G has *characteristic p* if $C_G(O_p(G)) \leq O_p(G)$.
- G has *local characteristic p* if p divides $|G|$ and all p-local subgroups of G have local characteristic p.

Notation

From now on p is prime, G is a finite $\mathcal{K}_p$-group of local characteristic p with $O_p(G) = 1$ and S is a Sylow p-subgroup of G.

Goal

Understand and classify the finite groups of local characteristic p with $O_p(G) = 1$.

Disclaimer

For p odd we do not expect to be able to achieve a complete classification. Some groups with a relatively small p-local structure will remain unclassified.

Definition

Let L be a finite group. A p-reduced normal subgroup of L is an elementary abelian normal p-subgroup Y of L with

$$O_p(L/C_L(Y)) = 1.$$

Y_L is the largest p-reduced normal subgroup of L.

Notation

$\tilde{C}$ is a maximal p-local subgroup of G with $N_G(\Omega_1 Z(S)) \leq \tilde{C}$ and
$E = O^p\Big(F_p^*(C_{\tilde{C}}(Y_{\tilde{C}}))\Big)$

We now distinguish two cases:

- $\neg E!$ There exist two distinct maximal p-local subgroups M_1 and M_2 with $E \leq M_1 \cap M_2$.
- $E!$ $\tilde{C}$ is the unique maximal p-local subgroup of G containing $\tilde{C}$.

The $\neg E!$-case

In the $\neg E!$ we choose suitable subgroups L_1 and L_2 with

$$E \leq L_1 \cap L_2 \quad \text{and} \quad O_p(\langle L_1, L_2 \rangle) = 1.$$

We then use the amalgam method to determine the structure of L_1 and L_2. Given L_1 and L_2 one should be able to identify G up to isomorphism.

If $\tilde{C}$ is the unique maximal p-local subgroup of G containing S, then either $\tilde{C}$ is a strongly p-embedded subgroup of G or one can apply the local CGT-theorem to obtain a p-local subgroup of a very restricted structure. But we currently do not know whether this information will be enough to identify G.
To avoid this problem we will assume from now on that S is contained in at least two maximal p-local subgroups of G.

The E!-case

Definition

A p-subgroup Q of G is called large, if $C_G(Q) \leq Q$, $(Q = O_p(N_G(Q)))$ and

$$N_G(A) \leq N_G(Q) \text{ for all } 1 \neq A \leq C_G(Q)$$

Lemma

Suppose E lies in a unique maximal subgroup of G. Then $O_p(\tilde{C})$ is a large p-subgroup of G.

Theorem (Structure Theorem)

Let Q be a large p-subgroup of G and M be a p-local subgroup of G with $Q \leq S \leq G$ and $Q \ntrianglelefteq M$. Put $M^\circ = \langle Q^M \rangle$, $\overline{M} = M/C_M(Y_M)$ and $I = [Y_M, M^\circ]$.
Suppose that $Y_M \leq Q$. Then one the following holds.

- $\overline{M^\circ} \cong SL_n(q), Sp_{2n}(q)$ *or* $Sp_4(2)'$ *and I is the corresponding natural module.*
- *There exists a normal subgroup K of $\overline{M}$ such that*
  - $K = K_1 \times \cdots \times K_r$, $K_i \cong Sl_2(q)$ *and*

    $$Y_M = V_1 \times \cdots \times V_r$$

    where $V_i := [Y_M, K_i]$ is a natural K_i-module.
  - *Q permutes the K_i's transitively.*
- *There exists a p-local subgroup M^* of G with $M \leq M^*$ and M^* fulfills the previous case.*

Suppose that $Y_M \not\leq Q$. Then one of the following holds:

- There exists a normal subgroup K of $\overline{M}$ such that $K = K_1 \circ K_2$ with $K_i \cong SL_{m_i}(q)$, $Y_M \cong V_1 \otimes V_2$ where V_i is a natural module for K_i and $\overline{M^\circ}$ is one of K_1, K_2 or $K_1 \circ K_2$.
- $(\overline{M^\circ}, p, I)$ is as given in the following table:

$\overline{M^\circ}$	p	I
$\mathrm{SL}_n(q)$	p	nat
$\mathrm{SL}_n(q)$	p	$\bigwedge^2(\mathrm{nat})$
$\mathrm{SL}_n(q)$	p	$\mathrm{S}^2(\mathrm{nat})$
$\mathrm{SL}_n(q^2)$	p	$\mathrm{nat} \otimes \mathrm{nat}^q$
$3\,\mathrm{Alt}(6), 3\,\mathrm{Sym}(6)$,	2	2^6
$\Gamma\mathrm{SL}_2(4), \Gamma\,\mathrm{GL}_2(4)$	2	nat
$\mathrm{Sp}_{2n}(q)$	2	nat
$\Omega_n^\pm(q)$	p	nat

$\overline{M^\circ}$	p	I
$\mathrm{O}_4^+(2)$	2	nat
$\Omega_{10}^\pm(q)$	2	spin
$\mathrm{E}_6(q)$	p	q^{27}
M_{11}	3	3^5
$2M_{12}$	3	3^6
M_{22}	2	2^{10}
M_{24}	2	2^{11}

Theorem (The H-Structure Theorem)

Suppose that Q is a large p-subgroup of G and let M be a p-local subgroup of G with $Q \leq S \leq G$ and $Y_M \not\leq Q$. Then there exists $H \leq G$ such that $M^\circ S \leq H$, $O_p(H) = 1$ and H has the same residual type as one of the following groups:

- *A group of Lie-type in characteristic p.*
- *For $p = 2$: $M_{24}, He, Co_2, Fi_{22}, Co_1, J_4, Fi_{24}, Suz, B, M, U_4(3)$ or $G_2(3)$.*
- *For $p = 3$: Fi_{24}, Co_3, Co_1 or M.*

Let $Q = O_p(\tilde{C})$. For $L \leq G$ put $L^\circ = \langle Q^g \mid g \in G, Q^g \leq L \rangle$. In view of the H-structure theorem we assume from now on that $Y_M \leq Q$ for all p-local subgroups M of G with $S \leq M$.

Definition

A finite group L is p-**minimal** if a Sylow p-subgroup of L is contained in a unique maximal subgroup of L but is not normal in L.

Theorem (The P!-Theorem)

Let $P \leq G$ such that

(*) $S \leq P \leq G$, *P is p-minimal,* $O_p(P) \neq 1$ *and* $Q \ntrianglelefteq P$.

Put $P^* := P^\circ O_p(P)$ *and* $Z_0 := \Omega_1(Z(S \cap P^*))$. *Then*

- *Y_P is a natural $SL_2(p^m)$-module for P^*.*
- *Z_0 is normal in $\widetilde{C}$.*
- *Either P is unique with respect to (*) or $P \sim q^2 SL_2(q)$.*

Theorem (The $\tilde{P}$!-Theorem)

Suppose that there exists more than one subgroup $\tilde{P}$ of G such that $S \leq \tilde{P}$, $\tilde{P}$ is p-minimal, $\tilde{P} \not\leq N_G(P^\circ)$ and $O_p(M) \neq 1$, where $M = \langle P, \tilde{P} \rangle$.
Then $p = 3$ or 5 and $M^\circ \sim p^{3+3^*+3^*} SL_3(p)$ for any such $\tilde{P}$.

Theorem (The Isolated Subgroup Theorem)

Let H be a finite group, $T \in \mathrm{Syl}_p(H)$ and P^* be p-minimal subgroup of H with $T \leq P^*$. Put $Y = \langle O^p(P^*)^H \rangle$ and

$$L = \langle R \mid T \leq R \leq H, R \text{ is p-minimal}, R \neq P^*. \rangle$$

Suppose that $O_p(L) \not\leq O_p(P^*)$ and P^* is narrow. Then $Y/O_p(Y)$ is quasisimple.

Corollary

Put $Y = \langle O^p(\tilde{P})^{\tilde{C}} \rangle$. Then $Y/O_p(Y)$ is quasisimple.

Theorem (The Small World Theorem.)

Let G be a finite group of local characteristic p with $O_p(G) \neq 1$. Then one of the following holds.

1. *E is contained in at least two maximal p-local subgroups of G.*
2. *S is contained in a unique maximal p-local subgroup of G.*
3. *There exist p-minimal subgroups P_1 and P_2 of G with $S \leq P_1 \cap P_2$, $O_p(P_i) \neq 1$, $P_1 \leq ES$ and $O_p(\langle P_1, P_2 \rangle) = 1$.*
4. *There exists a p-local subgroup M of G with $S \leq M$ and $Y_M \not\leq Q$.*
5. *There exists a p-minimal subgroup P of G with $S \leq P$ such that $Y_P \leq Q$ and $\langle Y_P^{\tilde{C}} \rangle$ is not abelian.*

Theorem (The Rank 2 Theorem)

Suppose there exists p-minimal subgroups P_1 and P_2 of G with $S \leq P_1 \cap P_2$, $P_1 \leq ES$, $O_p(P_i) \neq 1$ and $O_p(\langle P_1, P_2 \rangle) = 1$. Then one of the following holds:

- (P_1, P_2) *is a weak BN-pair.*
- *The structure of P_1 and P_2 is as in one of the following groups.*
  - *For $p = 2$: $U_4(3).2^e$, $G_2(3).2^e$, $D_4(3).2^e$, $HS.2^e$, F_3, $F_5.2^e$ or Ru.*
  - *For $p = 3$: $D_4(3^n).3^e$, Fi_{23}, F_2.*
  - *For $p = 5$: F_2.*
  - *For $p = 7$: F_1.*

Theorem (Local Recognition of finite spherical buildigs)

Let Π be an irreducible spherical Coxeter diagram with index set I with $|I| \geq 2$ and let Δ and Δ^* be thick buildings with Coxeter diagram Π. Let c and c^* be chambers of Δ and Δ^* respectively. Suppose that for each edge $J = \{x, y\}$ of Π, there exists a special isomorphism ϕ_J from $\Delta_J(c)$ to $\Delta^*_J(c^*)$. Then there exists a special isomorphism from Δ to Δ^*.

Notation

Let F be a finite group, let L be a finite simple group of Lie type of rank at least 3 and let Δ be the associated spherical building, so $L = \mathrm{Aut}^\dagger(\Delta)$. Suppose as well the following:

- *Π is the Coxeter diagram of Δ and I is its index set.*
- *c is a fixed chamber in Δ.*
- *For $T \subseteq J \subseteq I$, $L_J = \mathrm{Aut}^\dagger(\Delta_J(c))$ and $L_{JT} = N_{L_J}(\Delta_T(c))$. Thus $L_{J\emptyset}$ is a Borel subgroup of L_J and L_{JT} is the parabolic subgroup of type Π_T of L_J containing $L_{J\emptyset}$.*
- *$\mathcal{D}$ is a set of subsets of I of size at least two. A subset J of I is called a $\mathcal{D}$-set if $J \subseteq D$ for some $D \in \mathcal{D}$.*
- *For each $D \in \mathcal{D}$, F_D is a subgroup of F, $\phi_D \colon F_D \to L_D$ is a homomorphism and K_D is its kernel.*
- *For $J \subseteq D \in \mathcal{D}$, $F_{DJ} = \phi_D^{-1}(L_{DJ})$, $B_D = F_{D\emptyset}$ and $H_{DJ} = O^p(O^{p'}(F_{DJ}))$.*
- *$B = \langle B_D \mid D \in \mathcal{D} \rangle$.*

Hypothesis

- *Each irreducible subset of I of size at most 2 is a $\mathcal{D}$-set.*
- *The homomorphism ϕ_D is surjective for each $D \in \mathcal{D}$.*
- *If $D, E \in \mathcal{D}$ and $i \in D \cap E$, then $H_{Di} = H_{Ei}$. Thus for $i \in I$ we can define $H_i = H_{Di}$, where $D \in \mathcal{D}$ with $i \in D$. For $J \subseteq I$, let $H_J = \langle H_j \mid j \in J \rangle$ and $P_J = H_J B$ (so $H_\emptyset = 1$ and $P_\emptyset = B$).*
- *If $D, E \in \mathcal{D}$ and $i \in D$ then B_E normalizes F_{Di}.*
- *If $i, j \in I$ and $\{i, j\}$ is not a $\mathcal{D}$-set, then $H_i H_j = H_j H_i$ and $H_i \neq H_j$.*
- $[K_D, F_D] \leq O_p(K_D)$ *for each $D \in \mathcal{D}$.*
- $F = \langle F_D \mid D \in \mathcal{D} \rangle$.
- $|O_p(B)| \geq |O_p(L_\emptyset)|$.
- $O_p(F) = 1$.
- *There exists $D \in \mathcal{D}$ with $C_F(O_p(K_D)) \leq O_p(K_D)$.*

Theorem (Local Recognition of Finite Groups of Lie-type)

Under the above Notation and Hypothesis

$$O^{p'}(F) \cong L.$$

Theorem

Let M be a maximal p-local subgroup of G with $S \leq G$ and $[Y_M, M] \nleq Q$. Suppose $H \leq G$ such that $M^{\circ}S \leq H$, $H = N_G(F^*(H))$, $F^*(H)$ is a simple group of Lie type in characteristic p and rank at least two and $H \cap \tilde{C}$ is not solvable. Then $N_G(A) \leq H$ for all $1 \neq A \trianglelefteq S$.

Theorem

Suppose $H \leq G$ such that $S \leq H$, $H = N_G(F^*(H))$, $F^*(H)$ is a simple group of Lie type in characteristic p and rank at least two and (if p is odd) $F^*(H) \not\cong PSL_3(p^a)$, and $C_H(z)$ is soluble for some $1 \neq z \in Z(S)$. Then one of the following holds:

- $N_G(Q) = N_H(Q)$;
- $p = 2$ and $F^*(G) \cong Mat_{11}, Mat_{23}, G_2(3)$ or $P\Omega_8^+(3)$; or
- $p = 3$ and $F^*(G) \cong PSU_6(2), F_4(2), {}^2E_6(2), McL, Co2, Fi_{22}, Fi_{23}$ or F_2.

Theorem

Suppose that p is an odd prime and H is a strongly p-embedded subgroup of the finite group F. If $F^*(H)$ is a group of Lie type in characteristic p of rank at least two, then $F^*(H) \cong L_3(p)$.

The following groups have been characterized by their p-local structure:

p	G
2	$\mathrm{Aut}(G_2(3))$
2	$\Omega_8^+(3)$
3	Mat_{12}
3	$SL_3(3)$
3	$\Omega_8^+(2)$
3	$Fi_{22}, Fi_{23}, Fi_{24}, Fi'_{24}$
3	Co_3
3	$U_6(2)$

p	G
3	$Alt(8)$
3	McL
3	F_2
3	Co_1
3	$F_4(2)$
3	$E_6(2)$
5	Ly
$3, 5, 7$	F_1

Let H be a finite group and V finite dimensional $\mathbb{F}_p H$-module

Definition

Let A be a subgroup of H such that $A/C_A(V)$ is an elementary abelian p-group. A is a best offender of H on V if $|B| \cdot |C_V(B)| \leq |A| \cdot |C_V(A)|$ for every $B \leq A$.

Definition

The normal subgroup of H generated by the best offenders of H on V is denoted by $J_H(V)$.

A $\mathrm{J}_H(V)$- component is non-trivial subgroup K of $\mathrm{J}_H(V)$ minimal with respect to $K = [K, \mathrm{J}_H(V)]$.

Theorem (FF-Module Theorem, Guralnick-Malle)

Let M be a finite group with $F^*(M)$ quasisimple and V a faithful simple $\mathbb{F}_p M$-module. Suppose that $M = J_M(V)$.
Then (M, p, V) is one of the following:

M	p	V
$SL_n(q)$	p	nat
$Sp_{2n}(q)$	p	nat
$SU_n(q)$	p	nat
$\Omega_n^\epsilon(q)$	p	nat
$O_{2n}^\epsilon(q)$	2	nat
$G_2(q)$	2	q^6
$SL_n(q)$	p	$\bigwedge^2(\text{nat})$

M	p	V
$Spin_7(q)$	p	Spin
$Spin_{10}^+(q)$	p	Spin
$3.\,Alt(6)$	2	2^6
$Alt(7)$	2	2^4
$Sym(n)$	2	nat
$Alt(n)$	2	nat

Theorem (J-Module Theorem)

Let M be a finite $\mathcal{CK}$-group, V a faithful, reduced $\mathbb{F}_p M$-module. Put $J = \mathrm{J}_V(M)$ and let $\mathcal{J} = \mathcal{J}_V(M)$ be the set of J_V-components of V. Put $W = [V, \mathcal{J}]C_V(\mathcal{J})/C_V(\mathcal{J})$ and let $K \in \mathcal{J}$.

- *K is either quasisimple or $p = 2$ or 3 and $K \cong SL_2(p)'$.*
- *$[V, K, L] = 0$ for all $K \neq L \in \mathcal{J}$.*
- *$W = \bigoplus_{K \in \mathcal{J}} [W, K]$.*
- *$J^p J' = O^p(J) = \mathrm{F}^*(J) = \times\, \mathcal{J}$.*
- *W is a semisimple $\mathbb{F}_p J$-module.*

Theorem (J-Module Theorem, continued)

Let $J_K = J/C_J([W, K])$. Then $K \cong O^p(J_K)$ and one of the following holds:

- *$[W, K]$ is a simple K-module and $(J_K, [W, K])$ fullfills the assumptions and so also the conclusion of FF-Module Theorem*
- *J_K and $[W, K]$ are as follows (where N denotes a natural module and N^* its dual):*

J_K	$[W, K]$	*conditions*
$\mathrm{SL}_n(q)$	$N^r \oplus N^{*s}$	$\sqrt{r} + \sqrt{s} \leq \sqrt{n}$
$\mathrm{Sp}_{2n}(q)$	N^r	$r \leq \frac{n+1}{2}$
$\mathrm{SU}_n(q)$	N^r	$r \leq \frac{n}{4}$
$\Omega_n^\epsilon(q)$	N^r	$r \leq \frac{n-2}{4}$
$\mathrm{O}_{2n}^\epsilon(q)$	N^r	$p = 2, r \leq \frac{2n-2}{4}$

Definition (The Fitting Submodule)

Let $\mathbb{F}$ be a field, H a finite group and V a finite dimensional $\mathbb{F}H$-module.

- $\mathrm{rad}_V(H)$ is the intersection of the maximal $\mathbb{F}H$-submodules of V
- Let W be an $\mathbb{F}H$ submodule of V and $N \trianglelefteq H$. Then W is N-quasisimple if W is H-reduced, $W/\mathrm{rad}_W(H)$ is simple for $\mathbb{F}H$, $W = [W, N]$ and N acts nilpotently on $\mathrm{rad}_W(H)$.
- $\mathrm{S}_V(H)$ is the sum of all simple $\mathbb{F}H$-submodules of V.
- $\mathrm{E}_H(V) := \mathrm{C}_{\mathrm{F}^*(H)}(\mathrm{S}_V(H))$.
- W is a component of V if either W is a simple $\mathbb{F}H$-submodule with $[W, \mathrm{F}^*(H)] \neq 0$ or W is an $\mathrm{E}_H(V)$-quasisimple $\mathbb{F}H$-submodule.
- The Fitting submodule $\mathrm{F}_V(H)$ of V is the sum of all components of V.
- $\mathrm{R}_V(H) := \sum \mathrm{rad}_W(H)$, where the sum runs over all components W of V

Theorem

- *The Fitting submodule* $\mathrm{F}_V(H)$ *is* H*-reduced.*
- $\mathrm{R}_V(H)$ *is a semisimple* $\mathbb{F}\,\mathrm{F}^*(H)$*-module.*
- $\mathrm{R}_V(H) = \mathrm{rad}_{\mathrm{F}_V(H)}(H)$.
- $\mathrm{F}_V(H)/\mathrm{R}_V(H)$ *is a semisimple* $\mathbb{F}H$*-module*

Theorem

Let V *be faithful and* H*-reduced. Then also* $\mathrm{F}_V(H)$ *and* $\mathrm{F}_V(H)/\mathrm{R}_V(H)$ *are faithful and* H*-reduced.*

Nearly Quadratic Modules

Definition

Let $\mathbb{F}$ be a field, A a group and V an $\mathbb{F}A$-module. Then V is a *nearly quadratic* $\mathbb{F}A$-module (and A acts *nearly quadratically* on V) if $[V, A, A, A] = 0$ and
$[V, A] + C_V(A) = [v\mathbb{F}, A] + C_V(A)$ for every $v \in V \setminus [V, A] + C_V(A)$.

Theorem

Let $\mathbb{F}$ be field, H a group and V be a faithful semisimple $\mathbb{F}H$-module. Let $\mathcal{Q}$ be the set of nearly quadratic, but not quadratic subgroups of H. Suppose that $H = \langle \mathcal{Q} \rangle$. Then there exists a partition $(\mathcal{Q}_i)_{i \in I}$ of $\mathcal{Q}$ such that

- $H = \bigoplus_{i \in I} H_i$, *where* $H_i = \langle \mathcal{Q}_i \rangle$.
- $V = C_V(H) \oplus \bigoplus_{i \in I} [V, H_i]$.
- *For each* $i \in I$, $[V, H_i]$ *is a simple* $\mathbb{F}H_i$*-module.*

Theorem

Let H be a finite group, and V a faithful simple $\mathbb{F}_pH$-module. Suppose that H is generated by nearly quadratic, but not quadratic subgroups of H. Let W a Wedderburn-component for $\mathbb{F}_p\,\mathrm{F}^*(H)$ in V and $\mathbb{K} := \mathrm{Z}(\mathrm{End}_{\mathrm{F}^*(H)}(W))$. Then W is a simple $\mathbb{F}_p\mathbb{F}^*(H)$-module and one of the following holds for $H, V, W, \mathbb{K}$ and (if $V = W$) $H/C_H(\mathbb{K})$

H	V	W	$\mathbb{K}$	$H/C_H(\mathbb{K})$	
$(C_2 \wr \mathrm{Sym}(m))'$	$\mathbb{F}_3^m$	$\mathbb{F}_3$	$\mathbb{F}_3$	–	$m \geq 3, m \neq 4$
$\mathrm{SL}_n(\mathbb{F}_2) \wr \mathrm{Sym}(m)$	$(\mathbb{F}_2^n)^m$	$\mathbb{F}_2^n$	$\mathbb{F}_2$	–	$m \geq 2, n \geq 3$
$\mathrm{Wr}(\mathrm{SL}_2(\mathbb{F}_2), m)$	$(\mathbb{F}_2^n)^m$	$\mathbb{F}_2^n$	$\mathbb{F}_4$	–	$m \geq 2$
$\mathrm{Frob}(39)$	$\mathbb{F}_{27}$	V	$\mathbb{F}_{27}$	C_3	
$\Gamma\,\mathrm{GL}_n(\mathbb{F}_4)$	$\mathbb{F}_4^n$	V	$\mathbb{F}_4$	C_2	$n \geq 2$
$\Gamma\mathrm{SL}_n(\mathbb{F}_4)$	$\mathbb{F}_4^n$	V	$\mathbb{F}_4$	C_2	$n \geq 2$
$\mathrm{SL}_2(\mathbb{F}_2) \times \mathrm{SL}_n(\mathbb{F}_2)$	$\mathbb{F}_2^2 \otimes \mathbb{F}_2^n$	V	$\mathbb{F}_4$	C_2	$n \geq 3$
$3^{\cdot}\,\mathrm{Sym}(6)$	$\mathbb{F}_4^3$	V	$\mathbb{F}_4$	C_2	
$\mathrm{SL}_n(\mathbb{K}) \circ \mathrm{SL}_m(\mathbb{K})$	$\mathbb{K}^n \otimes \mathbb{K}^m$	V	*any*	1	$n, m \geq 3$
$\mathrm{SL}_2(\mathbb{K}) \circ \mathrm{SL}_m(\mathbb{K})$	$\mathbb{K}^2 \otimes \mathbb{K}^m$	V	$\mathbb{K} \neq \mathbb{F}_2$	1	$m \geq 2$
$\mathrm{SL}_n(\mathbb{F}_2) \wr C_2$	$\mathbb{F}_2^n \otimes \mathbb{F}_2^n$	V	$\mathbb{F}_2$	1	$n \geq 3$
$(C_2 \wr \mathrm{Sym}(4))'$	$\mathbb{F}_3^4$	V	$\mathbb{F}_3$	1	
$\mathrm{SU}_3(2)'$	$\mathbb{F}_4^3$	V	$\mathbb{F}_4$	1	
$\mathrm{F}^*(H) = \mathrm{Z}(H)K$ K *quasisimple*	?	V	?	1	

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