Complex Analysis: Exercise 3

1. What is the power series for the function $f(z) = 1/z$, centered at the point $z_0 = 1$? That is, let

$$\frac{1}{z} = \sum_{n=0}^{\infty} c_n (z-1)^n$$

for some neighborhood of the the complex number 1. What are the numbers c_n? What is the radius of convergence of this power series?

2. What is

$$\int_{|z|=1} \frac{e^z}{z^n} dz$$

for different values of $n \in \mathbb{N}$?

3. Let $f : \mathbb{C} \to \mathbb{C}$ be an entire function such that $|f(z)| \leq |z|^n$ for some $n \in \mathbb{N}$, and for all $z \in \mathbb{C}$. Show that f is a polynomial.