29c) Beh DUM $\Rightarrow$ NP Bew 77: $i \in$ Nullspieler $\Rightarrow \phi_i(v) = 0$

$i \in NSp \Rightarrow v(S \cup i) = v(S) \overset{S=\emptyset}{\Rightarrow} v(i) = 0 \overset{i\ NS}{\Rightarrow} v(S\cup i) - v(S) = v(i) = 0$

$\Rightarrow i \in DUM \Rightarrow \phi_i(v) = v(i) = 0 \Rightarrow i \in NSP$

29d) Beh ϕ hat DUM Bew $i \in DUM \Rightarrow \phi_i(v) = \frac{1}{n!} \sum_{\sigma \in S_n} \overbrace{v(P_\sigma(i) \cup i) - v(P_\sigma(i))}^{\overset{DUM}{=} v(i)}$

$= \underset{\substack{i \notin S \\ S \subset N}}{v(i)} \frac{1}{n!} \sum_\sigma 1 = v(i)$

30a) Betrachte $\sigma = \binom{i\ j}{j\ i}$ Transposition, $S \subset N$ $\boxed{1}$ $i,j \in S$ oder $i,j \notin S \Rightarrow$

$v^\sigma(S) = v(S) \; \boxed{6}\; i \in S, j \notin S \Rightarrow v^\sigma(S) = v(\sigma(S)) = v(S \setminus i \cup j) = v(S \cup j \setminus i)$
$\overset{\text{wie VL}}{=} v(S)$ Insgesamt $v^\sigma = v$ $\boxed{2}$ $\phi_i(v) = \phi_{\sigma(j)}(v) = (\phi(v))_{\sigma(j)} \overset{\sigma^*}{=} (\sigma^*(\phi(v)))_j$
$\overset{ANN}{=} (\phi(v^\sigma))_j \overset{\boxed{1}}{=} \phi(v)_j \Rightarrow$ SYM