# Probeklausur

Mittwoch, 31. Januar 2024 10:46

1a) $V$ endlich erzeugter VR, $M \subset V$ EZS von $V$ $\Rightarrow$

es gibt $L \subset M$ so dass $L$ Basis von $V$ ist

1bi) In den Spalten der Matrix stehen die Koeffizienten der

Bilder der Standardbasis unter $f$ bzgl. der Standardbasis

$$A = \begin{pmatrix} -1 & 3 & 2 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 2 & -1 & 1 \end{pmatrix}$$

1bii)

$$\begin{matrix} -1 & 0 & 1 & 2 \\ 3 & 1 & 0 & -1 \\ 2 & 1 & 1 & 1 \end{matrix} \quad \cdot 3 \;\; \cdot 2 \quad \mapsto \quad \begin{matrix} -1 & 0 & 1 & 2 \\ 0 & 1 & 3 & 5 \\ 0 & 1 & 3 & 5 \end{matrix} \quad \mapsto \quad \begin{matrix} -1 & 0 & 1 & 2 \\ 0 & 1 & 3 & 5 \\ 0 & 0 & 0 & 0 \end{matrix}$$

$$\Rightarrow \text{rang } f = 2 \overset{\text{Difo}}{\Rightarrow} \dim \ker = 3 - 2 = \underline{1}$$

1c) $\vec{0} = \lambda \vec{a} + \mu \vec{b} + \gamma \vec{c} \Rightarrow -\lambda + \mu = 0, \ \lambda + 2\gamma = 0$

$2\mu + 2\gamma = 0 \Rightarrow \mu = \lambda = -2\gamma = 2\mu \Rightarrow \mu = 0$

$\Rightarrow \mu = \lambda = \gamma = 0$

2a)

$$\left(\begin{array}{cccc|c} 1 & -1 & 2 & 0 & 1 \\ -1 & 1 & -1 & 1 & s \\ 2 & -2 & -1 & -5 & 2 \end{array}\right) + \xrightarrow{E_{21}(1)} \left(\begin{array}{cccc|c} 1 & -1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 & s+1 \\ 2 & -2 & -1 & -5 & 2 \end{array}\right) (-2)$$

$$\xrightarrow{E_{31}(-2)} \left(\begin{array}{cccc|c} 1 & -1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 & s+1 \\ 0 & 0 & -5 & -5 & 0 \end{array}\right) \cdot 5 \xrightarrow{E_{32}(5)} \left(\begin{array}{cccc|c} 1 & -1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 1 & s+1 \\ 0 & 0 & 0 & 0 & 5s+5 \end{array}\right)$$

$= A'$ (coefficient part)

$$\xrightarrow{V_2^4} \left(\begin{array}{cccc|c} 1 & 0 & 2 & -1 & 1 \\ 0 & 1 & 1 & 0 & s+1 \\ 0 & 0 & 0 & 0 & 5s+5 \end{array}\right) =: b'$$

$$\longmapsto \quad \begin{array}{cccc|c} 0 & 1 & 1 & 0 & s+1 \\ 0 & 0 & 0 & 0 & 5s+5 \end{array} =: b'$$

$$A'' = E_{32}(5)\, E_{31}(-2)\, E_{21}(1)\, A\, V_2^4$$

2b) $5s+5 = 0 \iff s = -1$

2c) $\mathbb{L}_{A''|0} = \left\{ \lambda_1 \begin{pmatrix} 2 \\ 1 \\ -1 \\ 0 \end{pmatrix} + \lambda_2 \begin{pmatrix} -1 \\ 0 \\ 0 \\ -1 \end{pmatrix} \;\middle|\; \lambda_1, \lambda_2 \in \mathbb{R} \right\}$

2d) $\mathbb{L}_{A|b} = \mathbb{L}_{A'|b'} \overset{s=-1}{=} V_2^4 \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + V_2^4\, \mathbb{L}_{A''|0}$

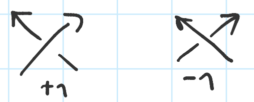
$= \left\{ \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \end{pmatrix} + \lambda_1 \begin{pmatrix} 2 \\ 0 \\ -1 \\ 1 \end{pmatrix} + \lambda_2 \begin{pmatrix} -1 \\ -1 \\ 0 \\ 0 \end{pmatrix} \;\middle|\; \lambda_1, \lambda_2 \in \mathbb{R} \right\}$

3a) ① $\langle \times \rangle = A \langle )( \rangle + A^{-1} \langle = \rangle$

② $\langle O \cup D \rangle = (-A^2 - A^{-2}) \langle D \rangle$

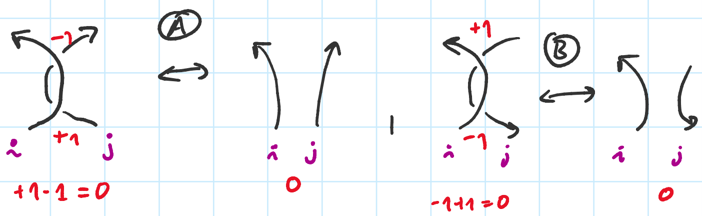
③ $\langle O \rangle = 1$

3b)



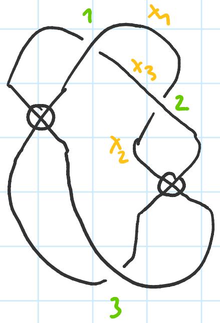

3c) $w(D) = \sum_{\substack{c \text{ klassische} \\ \text{Kreuzung von } D}} \varepsilon(c)$

3d) <u>Zwei Typen</u>




Falls Komponente i oder j nicht an RM II beteiligt ist, ist Summe der Kreuzungsinfos Null. Falls j über i liegt ist Summe Kreuzinfos Null.

4a)




1 $\quad x_3 + x_2 \equiv 2x_1 \mod 6$

2 $\quad x_1 + x_2 \equiv 2x_3 \mod 6$

3 $\quad x_1 + x_3 \equiv 2x_2 \mod 6$

LGS mod 6 :

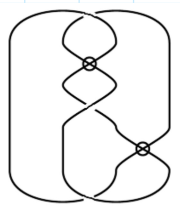
$$\begin{array}{ccc} x_1 & x_2 & x_3 \\ -2 & 1 & 1 \\ 1 & 1 & -2 \\ 1 & -2 & 1 \end{array} \mapsto \begin{array}{ccc} 0 & 3 & -3 \\ 1 & 1 & -2 \\ 0 & -3 & 3 \end{array}$$

$$\mapsto \begin{array}{ccc} 0 & 3 & -3 \\ 1 & 1 & -2 \\ 0 & 0 & 0 \end{array} \begin{array}{c} \\ |\cdot 3 \\ \\ \end{array} \xmapsto{\text{mod } 6} \begin{array}{ccc} 0 & 3 & -3 \\ 3 & 3 & 0 \\ 0 & 0 & 0 \end{array}$$

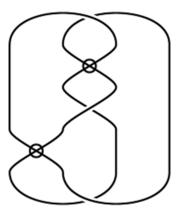
$$\Rightarrow \begin{array}{l} 3x_2 \equiv 3x_3 \mod 6 \\ 3x_2 \equiv 3x_1 \mod 6 \end{array} \overset{\text{mod } 6}{\Rightarrow} \begin{array}{l} x_2 = 0, \; x_3 = 2 \\ x_1 = 0 \end{array}$$

gesucht ist nicht-triviale Färbung

4b)



vertikal



horizontal

5a) $p$ Primzahl, $m \in \mathbb{N}$ : $\quad m^p \equiv m \mod p$

5b) Sei $(g_1, \dots, g_n) \in C$. Zz: $(g_1', g_2, \dots, g_n) \notin C$

Annahme $(g_1', \dots, g_n) \in C \Rightarrow g_1' g_2 \cdots g_n = c$

Annahme $(g_1', \ldots, g_n) \in C \Rightarrow g_1' g_2 \cdots g_n = c$

$\Rightarrow g_1' = c\, g_2^{-1} \cdots g_n^{-1} = g_1 \Rightarrow g_1' = g_1$ ↯ $\square$

5c)

$n = pq = 7 \cdot 11 = 77$, $\varphi(n) = 6 \cdot 10 = 60$

$ggT(60, 7) = 1 \Rightarrow e := 7$, $(7, 77)$ öS

$$7d \equiv 1\ (60) \Rightarrow \begin{aligned} 60 &= 7 \cdot 8 + 4 \\ 7 &= 4 \cdot 1 + 3 \\ 4 &= 3 \cdot 1 + 1 \end{aligned}$$

$\Rightarrow 1 = 4 - 3 = 4 - (7 - 4) = 2 \cdot 4 - 7 =$

$= 2 \cdot (60 - 7 \cdot 8) - 7 = 2 \cdot 60 - 7 \cdot 17$

$\Rightarrow d = -17 \equiv 43 \mod 60 \Rightarrow d = 43$

$\underline{m = 2}$: $2^7 \mod 77 \equiv 128 \equiv 51 \mod 77$

$\Rightarrow c = 51$

Entschlüsseln $51^{43} \mod 77$

5d) i) $C = \{ a_1 a_2 a_3 a_4 \mid a_1, a_2, a_3, a_4 \in \{0, \ldots, 7\}, 2a_1 + 3a_2 + 4a_3 + 5a_4 \equiv 0\ (8) \}$

$n = 4$, $q = 8$

ii) $8 + 15 + 24 + 5a_4 \equiv 0 \mod 8$

$\Rightarrow 5a_4 \equiv -47 \mod 8 \Rightarrow 5a_4 \equiv 1 \mod 8$

$\Rightarrow a_4 = 5 =$ Prüfziffer