

\boxed{A} f stetig partiell diff $\Rightarrow f$ tot diff $\Rightarrow f$ part. diff $+ f$ stetig

\Uparrow

Falle Richtungsableitungen

$$+ Df(a)(h) = \partial_a f(a)$$

\boxed{B} $D \subset \mathbb{R}^n$, $f: D \rightarrow \mathbb{R}^m \in C^1(D, \mathbb{R}^m)$

$\Leftrightarrow \frac{\partial}{\partial x_i} f_i: D \rightarrow \mathbb{R}$ stetig

$\Leftrightarrow f$ tot diff auf D und $f': D \rightarrow L(\mathbb{R}^n, \mathbb{R}^m)$

$$f'(x) = Df(x) = \begin{pmatrix} \partial_1 f_1(x) & \dots & \partial_n f_1(x) \\ \partial_1 f_2(x) & \dots & \partial_n f_2(x) \\ \vdots & & \vdots \\ \partial_1 f_m(x) & \dots & \partial_n f_m(x) \end{pmatrix} \text{ stetig}$$

□ Kettenregel:

$$\mathbb{R}^m \xrightarrow{f} \mathbb{R}^n \xrightarrow{g} \mathbb{R}^k$$

$$\mathbb{R}^m \xrightarrow{f'} L(\mathbb{R}^m, \mathbb{R}^n)$$

$$\mathbb{R}^n \xrightarrow{g'} L(\mathbb{R}^n, \mathbb{R}^k)$$

$$(g \circ f)' : \mathbb{R}^m \rightarrow L(\mathbb{R}^m, \mathbb{R}^k)$$

$$D(g \circ f)(\bar{x}) = \underbrace{Dg(f(\bar{x})) \cdot Df(\bar{x})}_{\text{darstellende Matrix}}$$

$$\in \mathbb{R}^n \rightarrow \mathbb{R}^k \quad \in \mathbb{R}^m \rightarrow \mathbb{R}^n$$

$$\in \mathbb{R}^k \rightarrow \mathbb{R}^m \quad \in \mathbb{R}^m \rightarrow \mathbb{R}^n$$

$$D(g \circ f)(\bar{x})(\bar{h}) = \underbrace{Dg(f(\bar{x}))}_{\mathbb{R}^n \rightarrow \mathbb{R}^k} \cdot \underbrace{Df(\bar{x})(\bar{h})}_{\in \mathbb{R}^n}}_{\text{als Abbildung}}$$

$$\in \mathbb{R}^m$$

$$\mathbb{R}^n \rightarrow \mathbb{R}^k$$

$$\in \mathbb{R}^n$$