Aufgabe 52

$\gamma: \mathbb{R} \to \mathbb{R}^3 \qquad \gamma(t) := (\cos 2\pi t, \sin 2\pi t, t) = (x,y,z)$

$f: \mathbb{R}^3 \to \mathbb{R} \qquad f(x,y,z) := \exp(xy+z)$

$$(f\circ\gamma)'(t) = \langle \operatorname{grad} f(\gamma(t)), \gamma'(t)\rangle = \left(y e^{xy+z}, x e^{xy+z}, e^{xy+z}\right)\Big|_{\left(\begin{smallmatrix}x\\y\\z\end{smallmatrix}\right)=\gamma(t)} \cdot \begin{pmatrix} -2\pi \sin 2\pi t \\ 2\pi \cos 2\pi t \\ 1 \end{pmatrix}$$

$$= y e^{xy+z}(-2\pi \sin 2\pi t) + x e^{xy+z}(2\pi \cos 2\pi t) + e^{xy+z}$$

$$= \left(-2\pi \underbrace{\sin^2(2\pi t)}_{=1-\cos^2 2\pi t} + 2\pi \cos^2(2\pi t) + 1\right) e^{xy+z} = \left(-2\pi + 2\pi \cos^2 2\pi t + 2\pi \cos^2 2\pi t + 1\right) e^{xy+z} = \left(1 - 2\pi + 4\pi \cos^2(2\pi t)\right) e^{xy+z}$$