Aufgabe 60 a $f(x,y,z) = (x^3+y^3+z)\,e^{z^2} = x^3 e^{z^2} + y^3 e^{z^2} + z e^{z^2}$

$f_x = 3x^2 e^{z^2}$, $f_y = 3y^2 e^{z^2}$, $f_z = e^{z^2} + (x^3+y^3+z)\,e^{z^2}\cdot 2z$

$= e^{z^2}(1 + 2zx^3 + 2zy^3 + 2z^2)$, $f_{xx} = 6x e^{z^2}$, $f_{xy} = 0$, $f_{xz} = 6x^2 z e^{z^2}$

$f_{yy} = 6y e^{z^2}$, $f_{yz} = 6zy^2 e^{z^2}$, $f_{zz} = e^{z^2}\cdot 2z(1+2zx^3+2zy^3+2z^2)$

$+ e^{z^2}(2x^3+2y^3+4z) \Rightarrow \operatorname{grad} f(0,0,0) = (f_x(0,0,0), f_y(0,0,0), f_z(0,0,0))$

$= (0,0,1)$, $Hf(0,0,0) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$

$T_1 f((x,y,z),(0,0,0)) = f(0,0,0) + f'(0,0,0)\cdot\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 0 + z = z$

$T_2 f((x,y,z),(0,0,0)) = T_1 f((x,y,z),(0,0,0)) + \frac{1}{2}(x,y,z)\,Hf(0,0,0)\begin{pmatrix} x \\ y \\ z \end{pmatrix} = z$

$T_3 f((x,y,z),(0,0,0)) = z + \sum_{|\alpha|=3} \frac{1}{\alpha!}\partial^\alpha f(0,0,0)\begin{pmatrix} x \\ y \\ z \end{pmatrix}^\alpha = z + \frac{1}{3!}6x^3 + \frac{1}{3!}6y^3 + \frac{1}{3!}6z^3$

$|\alpha| = 3 \Leftrightarrow \{(\alpha_1,\alpha_2,\alpha_3) = (3,0,0), (0,3,0), (0,0,3), (2,1,0), (2,0,1), (0,2,1), (0,1,2), (1,2,0)$
$(1,0,2), (1,1,1)\}$

Aufgabe 6a b $g: \mathbb{R}^2 \to \mathbb{R}$ $g(x,y) = x^y = e^{y \ln x}$

$g_x = y x^{y-1}$, $g_y = x^y \cdot \ln x$, $g_{xy} = x^{y-1} + y \cdot x^{y-1} \ln x = x^{y-1}(1 + y \ln x)$

$g_{yx} = y x^{y-1} \ln x + x^{y-1} = x^{y-1}(y \ln x + 1)$, $g_{xx} = y(y-1) x^{y-2}$,

$g_{yy} = x^y \cdot (\ln x)^2$, $g_{xxx} = y(y-1)(y-2) x^{y-3}$, $g_{yyy} = x^y (\ln x)^3$

$g_{xxy} = (2y-1) x^{y-2} + y(y-1) x^{y-2} \cdot \ln x$

$g_{yyx} = y x^{y-1} (\ln x)^2 + x^y \, 2(\ln x) \frac{1}{x} = x^{y-1} \ln x \, (y \cdot \ln x + 2)$

$T_1 f((x,y),(1,1)) = f(1,1) + f'(1,1) \begin{pmatrix} x-1 \\ y-1 \end{pmatrix} = 1 + \underbrace{g_x(1,1)}_{1}, \underbrace{g_y(1,1)}_{0} \begin{pmatrix} x-1 \\ y-1 \end{pmatrix}$

$= 1 + x - 1 = x$

$T_2 f((x,y),(1,1)) = x + \frac{1}{2} (x-1, y-1) \, Hf(1,1) \begin{pmatrix} x-1 \\ y-1 \end{pmatrix} = x + \frac{1}{2} (x-1, y-1) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x-1 \\ y-1 \end{pmatrix}$

$= x + \frac{1}{2!} (x-1, y-1) \begin{pmatrix} y-1 \\ x-1 \end{pmatrix} = x + (x-1)(y-1)$

$T_3 f((x,y),(1,1)) = T_2 + \sum_{|\alpha|=3} \frac{1}{\alpha!} \partial^\alpha g(1,1) \begin{pmatrix} x-1 \\ y-1 \end{pmatrix}^\alpha$ $\quad \alpha = (3,0), (0,3), (2,1), (1,2)$

f_{xxx} f_{yyy} f_{xxy} f_{yyx}

$$= \uparrow \quad T_2 + \frac{1}{2!1!} g_{xxy}(1,1)\,(x-1)^2(y-1) = x + (x-1)(y-1) + \frac{1}{2}(x-1)^2(y-1)$$

nur $g_{xxy}(1,1) = +1$ ist nicht Null

$$= x + xy - x - y + 1 + \frac{1}{2}(x^2 - 2x + 1)(y-1)$$

$$= x + xy - x - y + 1 + \frac{1}{2}(x^2y - x^2 - 2xy + 2x + y - 1)$$

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